Tuning Systems Derived from Timbre, and Timbres Derived from Tuning Systems, as Realised in Electronic Music

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Introduction

Since the near standardisation of Western music to 12 tone equal temperament, experimentation with tuning systems has been impractical, from the aspects of both composition and performance. Along with instrument design and construction, the lack of standardised forms of notation that are easily readable by instrumentalists have been the main obstacles that have made the use of alternate tuning systems a task attempted by only a few of the most dedicated and zealous of composers, such as Harry Partch[1]. Partch not only composed with alternate tuning systems (in his case a 43 pitch extended form of just intonation); he also built his own instruments and performed his own music.

Current technology in electronic music, however, provides an efficient (i.e., quick, easy and relatively cheap) means of overcoming the main impracticalities involved with the use of alternate tuning systems. Electronically performed music does not require a standardised form of notation that is easily readable by instrumentalists, and the instruments to be used, being synthetic, exist mostly in a "virtual" (i.e., software) environment. Therefore, experiments with tuning can be carried out simply by specifying certain parameters, rather than rebuilding instruments, or retraining musicians.

Whether alternate or conventional, the tuning systems that have been used in both acoustically and electronically realised Western music have been limited mostly to those that can provide consonant intervals and chords for timbres comprised of harmonically related partials. While most acoustically produced timbres comprise harmonically related

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[1] Harry Partch, *Genesis of a Music: An Account of a Creative Work, Its Roots and Its Fulfillment*, New York, Da Capo Press, 1974.

partials, not all of them do, and synthetic timbres (i.e., electronically produced) may be comprised of partials that bear *any* desired relationship.

This thesis will describe the development of tuning systems (or, more accurately in some cases, tuning *structures*) that are directly related to timbres that may contain non-harmonically related partials. Such tuning systems can provide consonant (or deliberately dissonant) intervals and chords for such timbres. This paper will also explore the concept of specifying the component frequencies of timbre for the purpose of achieving consonant or dissonant intervals and chords in any given tuning system, along with some procedures for actualizing related tuning systems and timbres in electronic music.

A brief description of the main tuning systems that have been used in Western Music

Pythagorean tuning, meantone temperament, and 12 tone equal temperament are the main tuning systems that have been used throughout the history of Western music. Each has its own deficiencies, and in part each has arisen to address the deficiencies of another.

Pythagorean tuning

The following steps achieve Pythagorean tuning by calculating the frequency of each chromatic pitch within an octave in a cycle of fifths:

1. Multiplying the frequency of the first pitch (for instance A at 440 Hz) by the ratio 3:2 gives the frequency of E (a perfect fifth above A 440 Hz) as 660 Hz.

2. Multiplying this frequency (660 Hz) by the ratio 3:2 gives the frequency of B (a perfect fifth above E 660 Hz) as 990 Hz.

3. Multiplying this frequency (990 Hz) by the ratio 1:2 gives the frequency of B (a whole tone above A 440 Hz) as 495 Hz.

Applying a similar procedure for each new fifth yields the frequency of each chromatic pitch within the octave above A at 440 Hz. These frequencies can simply be halved repeatedly to obtain the frequencies in consecutive lower octaves, or doubled repeatedly to obtain the frequencies in consecutive upper octaves. Appendix I shows more detail of Pythagorean tuning.

1/4 comma meantone temperament

The main "deficiency" of Pythagorean tuning is that the third diatonic major scale degree is very sharp in comparison to its occurrence in the harmonic series of the tonic. 1/4 comma meantone temperament remedies this deficiency by lowering each perfect fifth, as it is calculated in Pythagorean tuning, by 1/4 of what is known as a *syntonic* comma[2]. A syntonic comma is the difference between a major third in Pythagorean tuning, and a major third as it occurs in the harmonic series, expressed as a ratio of 81:80. As the third scale degree is four steps around a cycle of fifths from the tonic, the whole syntonic comma is accounted for when its pitch frequency is calculated, making it exactly as it occurs in the tonic's harmonic series. Although this makes the fifth diatonic scale degree flat in

[2] James Murray Barbour, *Tuning and Temperament A Historical Survey*, New York, Da Capo Press, 1972, p. V.

comparison to its occurrence in the harmonic series, the difference is only slight. Appendix II shows more detail of 1/4 comma meantone temperament.

12 tone equal temperament

The main "deficiency" of 1/4 comma meantone temperament is that it causes one of the fifths to be very sharp. This fifth would be between F and C if the frequencies had been calculated from A, and is known as the *wolf* fifth. 12 tone equal temperament remedies this deficiency by spacing each chromatic pitch equally throughout the octave, ensuring that any interval, regardless of which two pitches it occurs between, is exactly the same frequency ratio as it would be between any other two pitches of the same interval.

Calculating the frequency of each chromatic pitch within an octave in a cycle of semitones achieves 12 tone equal temperament. By multiplying the frequency of the first pitch (for instance A at 440 Hz) by 2 raised to the power of 1/12 (or $\sqrt[12]{2}$), the frequency of A# (a semitone above A 440 Hz) is calculated as 466.16 Hz (rounded off). Multiplying each new frequency by $2^{1/12}$ (or $\sqrt[12]{2}$) will yield the frequency of each chromatic pitch within the octave above A at 440 Hz. These frequencies can simply be halved repeatedly to obtain the frequencies in consecutive lower octaves, or doubled repeatedly to obtain the frequencies in consecutive upper octaves. Appendix III shows more detail of 12 tone equal temperament.

Just intonation

Just intonation has often been experimented with throughout the history of Western music, but has never been taken on as a standard for preferred use. Its popularity in experimental circles, and its relevance to the topic of this thesis justifies its inclusion here.

The main "deficiency" of 12 tone equal temperament is that the interval ratios (especially that of the major third, as with Pythagorean tuning) do not accurately reflect those of the harmonic series. Tonal harmony is an extremely important element of much Western music, and relies strongly on the interaction between tension and resolution, as manifest in the relative concepts of consonance and dissonance.

In the context of acoustics (as opposed to the context of harmony) an interval or chord may be considered consonant when the pitches within the interval or chord have numerous partials (i.e., component frequencies) in common. If there is significance in this acoustical definition of consonance, and if most timbres used in Western music are made up of harmonic partials (i.e., partials that occur within the harmonic series of a fundamental frequency), then a tuning system derived from the harmonic series would provide acoustically consonant intervals and chords, and hence more effective resolution of unstable harmonies.

There are many forms of just intonation, so only the most basic and common will be explained here. Calculating the pitch frequencies of three major triads, using the frequency ratios 5:4 for the major thirds, and 6:5 for the minor thirds, achieves a justly intoned diatonic major scale. The following steps calculate the pitch frequencies of a justly intoned major triad with A at 440 Hz as the tonic:

1. Multiplying the frequency of A (440 Hz) by the ratio 5:4 gives the frequency of C# (a major third above A 440 Hz) as 550 Hz.

2. Multiplying the frequency of C# (550 Hz) by the ratio 6:5 gives the frequency of E (a minor third above C# 550 Hz, and a perfect fifth above A 440 Hz) as 660 Hz.

Repeating these steps for the major triad on E at 660 Hz, and then reversing them for the major triad on D, starting with A at 440 Hz and using the reciprocal of each ratio (i.e., 5:6 and 4:5), yields the pitches required to make up a diatonic major scale on A 440 Hz. Any pitch frequencies outside of the octave above A 440 Hz can be derived simply by doubling or halving the pitch frequencies within the octave above A 440 Hz. Appendix IV shows more detail of just intonation.

The main advantage of just intonation is that each pitch of the tonic, subdominant and dominant triads has exactly the same frequency ratio to the root of the triad as it does in the harmonic series of the root of the triad. This means that virtually complete acoustical consonance is achieved by most major thirds and fifths sounded by timbres comprised of harmonically related partials. The main "deficiency" of just intonation is that this advantage is only true for one key. If a passage of music in just intonation was to modulate to a new key, the frequency ratios of the tonic, subdominant and dominant triads would no longer be as they occur in a corresponding harmonic series. Along with the wolf fifth of 1/4 comma meantone temperament, this deficiency can be remedied by equal temperament.

Chapter 1: Review of Literature in the Field of Alternate Tuning Systems in Electronic Music

Texts that provide a general overview of both conventional and alternate tuning systems

The Acoustical Foundations of Music, by John Backus[3], gives a concise and easily understandable description of the mathematical derivation of the main tuning systems that have been used in Western music: Pythagorean tuning, 1/4 comma meantone temperament, just intonation, and equal temperament. Backus demonstrates a strong disinclination to just intonation, partly because he attributes its popularity to a "numerological" perspective. It is unclear whether he condemns this perspective for its pertinence to superstition, or because he see no point in the manipulation of frequency ratios purely for the sake of their numerical properties. Backus's principal motive for denouncing just intonation is that the frequency ratios relative to the tonic change too much when a diatonic scale is started on a pitch different to that from which the frequencies were initially calculated. That is, the desire for consistent intervals is compromised by "remote" transpositions.

Tuning and Temperament, by James Murray Barbour[4], explains much of the terminology used in tuning system literature, and gives a comprehensive account of tuning systems throughout the history of Western music. It includes details of many varieties of Greek tetrachordal (4 pitch) tuning structures, meantone temperaments (such as 1/4

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[3] John Backus, *The Acoustical Foundations of Music*, New York, W.W. Norton and Company, 1977.

[4] James Murray Barbour, *Tuning and Temperament A Historical Survey*, New York, Da Capo Press, 1972.

comma, as well as 2/7, 1/3, 1/5, and 2/9 comma temperaments), extended and tempered forms of just intonation, and equal temperaments.

Tuning In: Microtonality in Electronic music, by Scott Wilkinson[5], gives a basic explanation of the mathematical concepts behind the construction of tuning systems, and gives a general history of the evolution of tuning systems. Wilkinson also provides simple descriptions of non-Western tuning systems, such as Indian, Indonesian, and Arabic, and gives details of how music in alternate tuning systems can be achieved on particular synthesizers that were commercially available at the time of the book's publication.

Literature regarding the exploration and realisation of alternate tuning systems in electronic music

Of the literature concerning the involvement of music technology with tuning systems, much is dedicated to supporting the use of equally tempered systems of 19, 31, or 53 tones per octave. The popularity of these equally tempered systems is due to their close approximations of justly intoned intervals (which are widely regarded as providing high acoustical consonance) while allowing modulation around a full cycle of key signatures without the interval ratios changing relative to the tonic (i.e., the desire for consistent intervals is *not* compromised by "remote" transpositions). Writers with this approach to tuning systems, assuming that their systems are to be used by timbres comprised of harmonic partials, can be said to be of an *acoustical* perspective. The primary concern of the acoustical perspective is the actual *sound* that results from the use of a particular tuning system.

[5] Scott R. Wilkinson, *Tuning In: Microtonality In Electronic Music*, Milwaukee, Hal Leonard Books, 1988.

Of the remaining literature, much is concerned with investigating the numerical properties of tuning systems. Although a considerable amount of this portion of the literature involves just interval ratios, their involvement is not so much for their acoustical qualities, but more for the theoretical curiosities that they present, such as "superparticular" ratios (frequency ratios in which the numerator is one less than the denominator) and prime limits. A "prime limit" is the largest prime number that can be divided into the numerator or denominator of all the frequency ratios between consecutive pitches in a just intonation system. Writers with this approach to tuning systems can be said to be of a *numerical* perspective.

The acoustical perspective and acoustical consonance

Writers in the field of electronic music who have supported equally tempered systems of 19, 31, or 53 tones per octave include Wendy Carlos[6], John Chalmers[7], Georg Hajdu[8], and M. Yunik and G. Swift[9]. Articles by all of these writers explain the use of a computer program that determines which equal divisions of the octave provide the most "consonant" intervals. While there are slight differences in the results of the program explained in each article, each comes to the similar conclusion that equally tempered

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[6] Wendy Carlos, "Tuning: At The Crossroads", *Computer Music Journal*, Vol. 11, no. 1, 1987, p. 29-43.

[7] John H. Chalmers Jr., "Construction and Harmonisation of Microtonal Scales in Non-12-Tone Equal Temperaments", *Proceedings of the 1982 International Computer Music Conference*, 1982, p. 534-541.

[8] Georg Hajdu, "17 Tones", *Proceedings of the 1992 International Computer Music Conference*, San Fransisco, ICMC and Department of Music, College of Humanities and the Arts, San Jose State University, 1992, p. 449-450.

[9] M. Yunik and G.W. Swift, "Tempered Music Scales for Sound Synthesis", *Computer Music Journal*, Vol. 4, no. 4, 1980, p. 60-65.

systems of 19, 31, or 53 tones per octave provide close approximations to what are commonly heard as the most consonantly tuned intervals.

The difference in their respective views concerns the question of which division yields the most consonant intervals. Carlos[10] finds equal divisions of 118, 65, 53, 31 and 19 tones per octave to give exact, or the closest approximations to, consonant intervals in that order. Figure 1-1 shows a graph of the results of Carlos's program. On the other hand, Hajdu[11], and Yunik and Swift[12] find the equal division of the octave into 19 tones to be the best, although they still find divisions of 31 and 53 to be acceptable. These differences are only due to variations in the definitions of consonance that have been used in each of the computer programs.

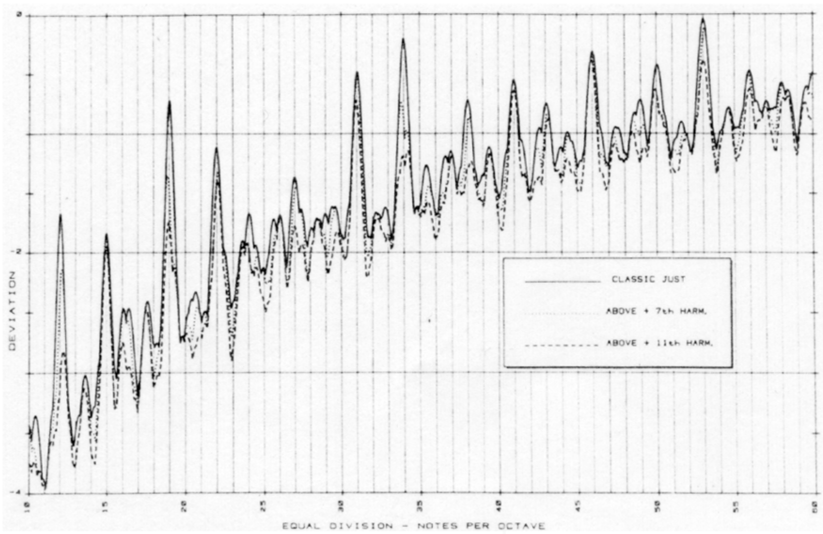

Figure 1-1. A diagram by Wendy Carlos[13]. The peaks of this graph show the number of equal divisions of the octave that provide relatively good approximations to justly intoned intervals. The higher the peak, the better the approximations.

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[10] Wendy Carlos, "Tuning: At The Crossroads", *Computer Music Journal*, Vol. 11, no. 1, 1987, p. 29-43.

[11] Georg Hajdu, "17 Tones", *Proceedings of the 1992 International Computer Music Conference*, San Fransisco, ICMC and Department of Music, College of Humanities and the Arts, San Jose State University, 1992, p. 449-450.

[12] M. Yunik and G.W. Swift, "Tempered Music Scales for Sound Synthesis", *Computer Music Journal*, Vol. 4, no. 4, 1980, p. 60-65.

[13] Wendy Carlos, "Tuning: At The Crossroads", *Computer Music Journal*, Vol. 11, no. 1, 1987, p. 29-43.

These definitions of consonance are all, to varying degrees, based on that of Helmholtz[14]. His concept of consonance was based on what is known as "roughness". This "roughness", caused by rapid phase addition and cancellation, is perceived as a *beating* between two pure (i.e., sine wave) tones, as the frequency of one approaches the other. When the frequencies of the two tones are far enough apart, no beating is perceived between the two tones. When the frequencies of the two tones are close, but not equal, a fast beating is perceived between the two tones. When the frequencies of the two tones are closer, but still not equal, a *slow* beating is perceived between the two tones. When the frequencies of the two tones are equal, *no* beating is perceived between the two tones. Sound Example 1a demonstrates the phenomenon of beating between two pure tones; the pitch of one tone remains fixed for the duration of the example as A at 440 Hz, while the other tone glides downwards from an interval of a justly intoned major third (above A 440 Hz) to unison (with A 440 Hz), and then back again.

According to Helmholtz, the "roughness" (or beating) between two pure tones of close, but not equal, frequency, can be construed as dissonance; hence consonance occurs when there is no roughness. Roughness occurs when the frequency difference between two tones is less than a minor third (approximately). This frequency difference in which roughness occurs is known as the "critical bandwidth". Through experimentation, Plomp and Levelt[15] found that the critical bandwidth varies across the audible spectrum (for psycho-acoustic reasons), but can generally be considered to be within an approximate minor third.

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[14] Herrman L.F. Helmholtz, *On The Sensations of Tone*, New York, Dover Publications, 1954.

[15] R. Plomp and W.J.M. Levelt, "Tonal Consonance and the Critical Bandwidth", *Journal of the Acoustical Society of America*, Vol. 38, 1965, p. 548-560.

The phenomenon of roughness can be heard between the pure tones that make up the complex waveforms of timbre. Intervals that are tuned such that the partials of simultaneously sounding timbres coincide at exactly the same frequencies will cause little or no roughness, and are therefore acoustically consonant. Sound Example 1b demonstrates this; the pitch of one complex tone remains fixed for the duration of the example as A at 440 Hz, while the other complex tone glides downwards from an interval of a justly intoned major sixth (above A 440 Hz) to a justly intoned fifth (above A 440 Hz), and then back again. The fundamentals of the two complex tones remain sufficiently separated for no beating to occur between them, yet beating still occurs due to the closeness in frequency of the partials of each tone. The beating is minimal (to the point of being negligible), when the interval between the two tones remains a justly intoned fifth, because many of the partials of each tone coincide at exactly the same frequencies. This is why exact, or at least close approximations to, just intervals are commonly desired for use with harmonic timbres.

Geary[16], Keislar[17], and Mathews and Pierce[18], have all conducted surveys on the relevance of Helmholtz's theory of consonance to the development of tuning systems. All surveys took into account the variance in the level of musical education in their subjects, and resulted in contrasting conclusions. While Geary's survey attests to Helmholtz's theory, Keislar's does not, and the survey by Mathews and Pierce was inconclusive.

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[16] J.M. Geary, "Consonance and Dissonance Pairs of Inharmonic Sounds", *Journal of the Acoustical Society of America*, Vol. 67, no. 5, 1980, p. 1785-1789.

[17] Douglas F. Keislar, "The Relevance of Beating Partials for Musical Intonation", *Proceedings of the 1991 International Computer Music Conference*, 1991, p. 202-205.

[18] Max V. Mathews and John R. Pierce, "Harmony and Nonharmonic Partials", *Journal of the Acoustical Society of America*, Vol. 68, 1980, p. 1252-1257, and Max V. Mathews and John R. Pierce, "Harmony and Nonharmonic Partials", *Rapports IRCAM*, 1980, no. 28.

The "numerical" perspective

Rather than finding equally tempered systems that approximate the supposedly ideal just frequency ratios, another approach by writers in the field of electronic music to the exploration of tuning systems is to either disregard just frequency ratios, or to expand the structure of tuning systems based on just frequency ratios by arranging these ratios with mathematical processes, such as set theory or symmetrical cycles.

An article by Gerald Balzano[19] describes how equally tempered divisions of the octave into 20, 30, and 42 tones are musically viable because they possess similar geometric properties with the 12 tone equal division of the octave. These geometric properties are obtained by arranging the pitch classes into a matrix , and then drawing lines to connect major and minor thirds, creating right angled triangles, joined together in such a way that major and minor (i.e., diatonic) chords are represented as parallelograms. Figures 1-2 and 1-3 show the similarity between 12 and 20 tone equal temperament in terms of this right angle triangle and parallelogram arrangement. As the frequency ratios between consecutive pitches in 12 tone equal temperament is $\sqrt[12]{2}$, the frequency ratio between each consecutive pitch in 20, 30, and 42 tone equal temperament is $\sqrt[20]{2}$, $\sqrt[30]{2}$, and $\sqrt[42]{2}$ respectively.

[19] Gerald J. Balzano, "The Group Theoretic Description of 12-Fold and Microtonal Pitch Systems", *Computer Music Journal*, Vol. 4, no. 4, 1980, p. 66-84.

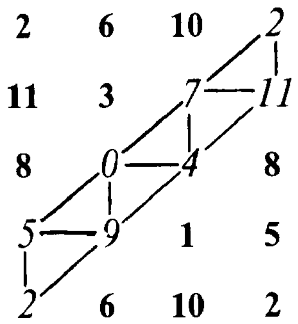

Figure 1-2. Part of a diagram by Gerald Balzano[20]. Each number represents a pitch class of 12 tone equal temperament, so vertical lines connect minor thirds, horizontal lines connect major thirds, and diagonal lines connect perfect fifths. Major and minor (i.e., diatonic) chords are therefore represented as right angle triangles and parallelograms.

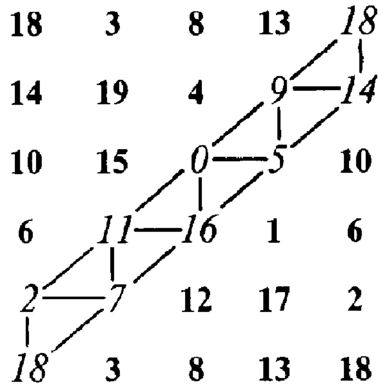

Figure 1-3. Part of a diagram by Gerald Balzano[21]. Each number represents a pitch class of 20 tone equal temperament, and the pattern can be seen to be similar to the one in figure 1-6, so the connected pitch classes are what could be construed as diatonic chords in 20 tone equal temperament.

An article by John Chalmers and Ervin Wilson[22] explains how groups of just frequency ratios are formed around points of symmetry to produce “combination product sets”. This article also describes 19 and 31 tone equally tempered divisions of a “stretched” or “shrunk” octave, meaning that the octave that is divided equally into 19 or 31 tones is actually slightly more or less than a “pure” octave (a “pure” octave being the frequency

[20] Gerald J. Balzano, “The Group Theoretic Description of 12-Fold and Microtonal Pitch Systems”, *Computer Music Journal*, Vol. 4, no. 4, 1980, p. 66-84.

[21] Gerald J. Balzano, “The Group Theoretic Description of 12-Fold and Microtonal Pitch Systems”, *Computer Music Journal*, Vol. 4, no. 4, 1980, p. 66-84.

[22] John H. Chalmers Jr. and Ervin M. Wilson, “Combination Product Sets and Other Harmonic and Melodic Structures”, *Proceedings of the 1981 International Computer Music Conference*, 1981, p. 348-354.

ratio of exactly 2:1). This achieves even closer approximations of, or exact, just frequency ratios of particular intervals. Tuning systems developed by Wendy Carlos[23], namely the alpha, beta, and gamma scales, can also be interpreted as equal divisions of stretched or shrunk octaves, although they were not derived in quite the same way.

An article by Gerald Lefkoff[24] demonstrates the generation of what he terms "tuned cyclic tone systems". Lefkoff repeatedly calculates the frequency of pitches by exponentiating and multiplying the variables of frequency ratios to generate recurring patterns of tuned tones. Changing the exponent, multiplier, or ratio results in new tunings.

An article by Max Mathews and John Pierce[25] illustrates the derivation and development of the Bohlen-Pierce, or "BP", scale. The BP scale is derived from the triadic frequency ratios 3:5:7 and 5:7:9, rather than the 4:5:6 ratio of the major triad in basic just intonation. The diatonic BP scale is then "tempered" equally to become a full chromatic scale of 13 tones per perfect twelfth. Therefore, the frequency ratio between each consecutive pitch is $3^{1/13}$ (or $\sqrt[13]{3}$). Although the BP scale is not derived from the *acoustical* perspective, Mathews and Pierce discuss the acoustical properties of the BP scale, and conclude that it would be most suitable for timbres with only odd harmonic partials. Also, a survey is conducted to find the most and least dissonant chords of the BP scale.

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23 Wendy Carlos, "Tuning: At The Crossroads", *Computer Music Journal*, Vol. 11, no. 1, 1987, p. 29-43.

24 Gerald Lefkoff, "Tuned Cyclic Systems", *Proceedings of the 1978 International Computer Music Conference*, Vol. 1, 1978, p. 226-245.

25 Max V. Mathews and John R. Pierce, "The Pierce-Bohlen Scale", in Max V. Mathews and John R. Pierce, (ed.), *Current Directions in Computer Music Research*, Cambridge Massachusetts, The MIT Press, 1989, p. 165-171.

The concept of tuning systems derived from the component frequencies of particular timbres

So far, it can be seen that much of the research in the field of tuning systems and electronic music has either been directed towards the realisation of exact or close approximations of just frequency ratios, or has at least involved just frequency ratios in developing a new or extended tuning system. This has usually been because just frequency ratios can represent musical intervals as they occur in the harmonic series.

The timbres of most Western instruments are made up of harmonic partials; that is, their component frequencies are whole number multiples of a fundamental frequency. As just frequency ratios are a reflection of those of the harmonic series, they can be shown to provide acoustically consonant intervals and chords when realised with these timbres. This is because acoustical consonance occurs when the partials of two (or more) simultaneously sounding timbres coincide at exactly the same frequencies. Therefore, if *non*-just frequency ratios were sounded with timbres that contain harmonic partials, or conversely, if just frequency ratios were sounded with timbres that contain *non*-harmonic partials, the result may not be acoustically consonant at all.

Wendy Carlos[26], James Dashow[27], William Sethares[28], Frank Slaymaker[29], and Ronald Smith[30] have all written on the concept of deriving tuning systems from the

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[26] Wendy Carlos, "Tuning: At The Crossroads", *Computer Music Journal*, Vol. 11, no. 1, 1987, p. 29-43.

[27] James Dashow, "Spectra as Chords", *Computer Music Journal*, Vol. 4, no. 1, 1980, p. 43-52, and James Dashow, "Three Methods for the Digital Synthesis of Chordal Structures with Non-Harmonic Partials", *Interface*, Vol. 7, 1978, p. 69-94.

[28] William A. Sethares, "Local Consonance and the Relationship Between Timbre and Scale", *Journal of the Acoustical Society of America*, Vol. 94, no. 3, 1993, p. 1218-1228, and William A. Sethares, "Relating Timbre and Tuning", *Experimental Musical Instruments*, Vol. 9, no. 2, 1993, p. 22-29.

[29] Frank H. Slaymaker, "Chords from Tones Having Stretched Partials", *Journal of the Acoustical Society of America*, Vol. 47, 1970, p. 1569-1571.

frequency relationships of partials to the fundamental frequencies of specific timbres. As just frequency ratios reflect the relationships between the harmonic partials of timbre, tuning systems can also be achieved by using frequency ratios that reflect those between the *non*-harmonic partials of a timbre. Acoustical consonance can be provided by intervals and chords sounded by the timbre comprised of the non-harmonic partials from which a tuning system's frequency ratios were derived. Carlos[31] developed a tuning system for the gamelan in this way.

Dashow[32] used synthesis methods such as amplitude modulation, frequency modulation, and "ring" modulation (the multiplication of two signals) to produce timbres of non-harmonic partials (i.e., component frequencies that are *not* whole number multiples of the fundamental frequency) in which the partials are audible as separate tones. Because the non-harmonic partials are audible as distinct tones, the tuning of the audible chords are intrinsically related directly to the timbre. Sethares[33] used the minimum points of "dissonance curves" (graphs of acoustical dissonance against intervallic frequency ratio, based on the experiments of Plomp and Levelt[34]) to create a tuning structure for use with the percussive sound of rocks from Chaco Canyon New Mexico.

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[30] Ronald Bruce Smith, "Formant Regions of Harmonic Spectra as a Model for Quarter-Tone Chord Voicings", *Proceedings of the 1991 International Computer Music Conference*, 1991, p. 322.

[31] Wendy Carlos, "Tuning: At The Crossroads", *Computer Music Journal*, Vol. 11, no. 1, 1987, p. 29-43.

[32] James Dashow, "Spectra as Chords", *Computer Music Journal*, Vol. 4, no. 1, 1980, p. 43-52, and James Dashow, "Three Methods for the Digital Synthesis of Chordal Structures with Non-Harmonic Partials", *Interface*, Vol. 7, 1978, p. 69-94.

[33] William A. Sethares, "Tunings for Nonharmonic Sounds", *Synaesthetica '94, Symposium on Computer Animation and Computer Music, Proceedings 1-3 July 1994*, Canberra, Australian Centre for thhe Arts and Technology, 1994.

[34] R. Plomp and W.J.M. Levelt, "Tonal Consonance and the Critical Bandwidth", *Journal of the Acoustical Society of America*, Vol. 38, 1965, p. 548-560.

Slaymaker[35] made adjustments to 12 tone equal temperament to suit timbres comprised of “stretched” and “shrunk” partials (i.e., partials whose frequencies are slightly more or slightly less than whole number multiples of a fundamental tone). Smith[36] used spectral analysis of timbre to develop a model for voicing chords in “quarter-tone” (i.e., 24 tone equal) temperament in relation to formant regions.

Pierce[37] and Sethares[38] have both written also on the concept of specifying the component frequencies of timbre for the purpose of achieving consonant intervals and chords in a given tuning system. Pierce[39] determined the component frequencies of a timbre by using multiples of the frequencies of an 8 tone equally tempered scale. Sethares[40] uses mathematical problem solving techniques to find the “optimal” number of partials, partial frequencies, and partial amplitudes to consonantly suit a given tuning system.

[35] Frank H. Slaymaker, “Chords from Tones Having Stretched Partials”, *Journal of the Acoustical Society of America*, Vol. 47, 1970, p. 1569-1571.

[36] Ronald Bruce Smith, “Formant Regions of Harmonic Spectra as a Model for Quarter-Tone Chord Voicings”, *Proceedings of the 1991 International Computer Music Conference*, 1991, p. 322.

[37] John R. Pierce, “Attaining Consonace in Arbitrary Scales”, *Journal of the Acoustical Society of America*, Vol. 40, 1966, p. 249.

[38] William A. Sethares, “Local Consonance and the Relationship Between Timbre and Scale”, *Journal of the Acoustical Society of America*, Vol. 94, no. 3, 1993, p. 1218-1228, and William A. Sethares, “Relating Timbre and Tuning”, *Experimental Musical Instruments*, Vol. 9, no. 2, 1993, p. 22-29.

[39] John R. Pierce, “Attaining Consonace in Arbitrary Scales”, *Journal of the Acoustical Society of America*, Vol. 40, 1966, p. 249.

[40] William A. Sethares, “Local Consonance and the Relationship Between Timbre and Scale”, *Journal of the Acoustical Society of America*, Vol. 94, no. 3, 1993, p. 1218-1228, and William A. Sethares, “Relating Timbre and Tuning”, *Experimental Musical Instruments*, Vol. 9, no. 2, 1993, p. 22-29.

The unaddressed issues

While much has been written on tuning systems from what has been termed the *acoustical* and *numerical* perspectives, and a considerable amount from the perspective of directly relating a tuning system to a particular timbre (which can also be seen as an extension of the acoustical perspective), there are two issues that have been given little or no attention in any of the literature.

First, no attention has been given to the exploration of a tuning system which is designed to maximise, instead of avoiding, the “roughness” between the component frequencies of simultaneous pitches. Surely, if the resolution of tonal harmony is dependent on the interaction between tension and resolution, a tuning system that provides maximum acoustical *dissonance* for a particular timbre is just as important as one that provides maximum consonance.

Second, very little attention has been given to the problem of developing a tuning system that provides acoustically consonant intervals for simultaneously sounded timbres that each have different relationships between their comprised component frequencies. For example, could a scale be found that provides acoustically consonant intervals for a duet of a wind instrument (for instance, an oboe, the timbre of which would typically contain harmonic component frequencies) and a metalphone (for instance, the crotales, the timbre of which would typically contain *non*-harmonic component frequencies)?

Possible solutions to these issues will be discussed in the course of this thesis, but the main focus will be to explore the possibilities of deriving tuning systems from the component frequencies of timbre, along with the converse possibility of deriving the component frequencies of timbre from given tuning systems. There is no intention to argue that deriving tuning systems from the component frequencies of timbre is the *only correct* *method* of intonation, or that any other method of intonation is, in any way, musically inappropriate.

Chapter 2: Timbre

The "component frequencies", or "partials", of timbre

Excluding the effects of "enveloping" (the variation of amplitude with time), the method of attack (such as bowing, plucking, blowing or striking), and pitch inflection (such as vibrato), timbre is determined by numerous pure tones of differing frequencies and amplitudes fused into a single entity.

The pure tones that combine to make up a timbre are sometimes referred to as "harmonics" or "overtones". As the term "harmonic" implies that the frequencies of the pure tones are all whole number multiples of a single frequency, and the term "overtone" implies that the pure tones are all higher in frequency than their corresponding fundamental tone, they are referred to in this thesis either as "component frequencies" or "partials". Generally, the term "partial" will be used where frequency and amplitude are concerned, and "component frequency" where only frequency is concerned.

The term "fundamental", or "fundamental tone", is used to denote the component frequency of a timbre that is perceived as its actual pitch. A fundamental tone often has the highest amplitude and lowest frequency of a timbre's partials, but not always. In some cases, a fundamental tone may not even exist outside of a listener's head, as its presence is constructed by the inner ear and brain. Frequencies that are perceived in this way are known as *heterodyne components*[41]. In cases where a heterodyne component constitutes a fundamental tone, its frequency can be that to which the most other existing component frequencies are whole number multiples. Hence, if a timbre consists of component

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[41] Arthur H. Benade, *Fundamentals of Musical Acoustics*, New York, Oxford University Press, 1976, p. 254.

frequencies that are whole number multiples of a frequency that is not comprised in the timbre, this frequency may still perceived as the fundamental.

Sometimes, especially where the component frequencies of a timbre are not whole number multiples of a fundamental, there may be ambiguity as to which component frequency is the fundamental tone, or a fundamental tone may barely be discernible at all. A timbre with either of these characteristics may be referred to as "indefinitely pitched" or "unpitched" respectively. A good example of an indefinitely pitched timbre is that of some clock chimes[42], in which the great difference in frequency and decay time between the loudest sounding partials can cause confusion as to which partial constitutes the actual pitch. A good example of an unpitched timbre is that of some cymbals[43], in which a multitude of high amplitude partials are close in frequency, causing the constitution of any one partial as the timbre's actual pitch to be, in most cases, inaudible.

Component frequencies of a timbre that are whole number multiples of the fundamental tone can be said to be "harmonically related" to the fundamental tone. Where this is the case, these component frequencies may be referred to as "harmonic component frequencies" or "harmonic partials". Where this is not the case, these component frequencies may be referred to as "non-harmonic component frequencies" or "non-harmonic partials".

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[42] Arthur H. Benade, *Fundamentals of Musical Acoustics*, New York, Oxford University Press, 1976, p. 52.

[43] Neville H. Fletcher and Thomas D. Rossing, *The Physics of Musical Instruments*, New York, Springer-Verlag, 1991, p. 556.

Harmonic timbres

A timbre can be referred to as “harmonic” if it contains only harmonic component frequencies. Harmonic component frequencies are whole number multiples of the frequency of a fundamental tone, so can be calculated in the following manner:

let f_n = the frequency of the nth partial

$f_1 = f_1$

$f_2 = 2f_1$

$f_3 = 3f_1$

$f_4 = 4f_1$

...

$f_n = nf_1$ equation 1

A timbre can be referred to as “non-harmonic” if it contains any non-harmonic partials. This general definition comprises two divisions: stretched and shrunk, and completely non-harmonic timbres.

Stretched or shrunk timbres

In comparison to harmonic timbres, the component frequencies of stretched or shrunk timbres, in terms of frequency difference, are spaced further apart (for stretched timbres), or closer together (for shrunk timbres), so can be calculated with a variation of equation 1:

let f_n = the frequency of the nth partial

S = index of stretching or shrinkage

$f_n = n^S f_1$ equation 2

If S is equal to 1 (or any positive whole number), equation 2 will generate component frequencies for a harmonic timbre. If S is greater than 1, but not a whole number, equation 2 will generate component frequencies for a stretched timbre. If S is less than 1, equation 2 will generate component frequencies for a shrunk timbre. For the sake of explanation, values of S less than 0 are not being considered here. Table 2-1 compares the component frequencies of a stretched timbre with those of a harmonic timbre, and table 2-2 compares the component frequencies of a shrunk timbre with those of a harmonic timbre.

Table 2-1. The first nine component frequencies of a stretched timbre (calculated with an S value of 1.261859507) as compared to those of a harmonic timbre.

Partial number (f_n)	**Stretched Component frequency (Hz)**	**Ratio to f_1**	**Harmonic Component frequency (Hz)**	**Ratio to f_1**
f_1	110	1	110	1
f_2	263.78	2.4	220	2
f_3	440	4	330	3
f_4	632.61	5.75	440	4
f_5	838.31	7.62	550	5
f_6	1055.12	9.59	660	6
f_7	1278.75	11.63	770	7
f_8	1516.9	13.79	880	8
f_9	1760	16	990	9

Table 2-2. The first nine component frequencies of a shrunk timbre (calculated with an S value of 0.79248125) as compared to those of a harmonic timbre.

Partial number (f_n)	Shrunk Component frequency (Hz)	Ratio to f_1	Harmonic Component frequency (Hz)	Ratio to f_1
f_1	110	1	110	1
f_2	190.52	1.73	220	2
f_3	262.68	2.39	330	3
f_4	330	3	440	4
f_5	393.8	3.58	550	5
f_6	455.07	4.14	660	6
f_7	514.14	4.67	770	7
f_8	571.56	5.2	880	8
f_9	627.55	5.71	990	9

The partials of the harmonic series may have to be stretched considerably before a stretched timbre is perceived as non-harmonic. The timbres given by strings under tension, such as those of a piano, have been shown to be stretched[44], although the piano is perceived as a harmonic timbre.

Completely non-harmonic timbres

A timbre may be referred to as “completely non-harmonic” if it mostly contains partials that are not in the harmonic series of its fundamental tone (if it has a fundamental tone), and if the relationship between the frequencies of the partials and fundamental tone does not resemble that of the harmonic series (as it does in a stretched or shrunk timbre). Completely non-harmonic timbres are often heard as indefinitely pitched or unpitched. Good examples of acoustically produced completely non-harmonic timbres are those of circular metal percussion instruments, such as a tam-tam or a cymbal. While there are mathematical relationships between the partials of many completely non-harmonic timbres that can be produced acoustically, they are beyond the scope of this thesis. Table 2-3 compares the component frequencies of a completely non-harmonic timbre with those of stretched, shrunk, and harmonic timbres.

[44] Arthur H. Benade, *Fundamentals of Musical Acoustics*, New York, Oxford University Press, 1976, p. 315.

Table 2-3. A comparison of the first 16 component frequencies present in the completely non-harmonic timbre of a circular plate with free edges[45], to those of the stretched, shrunk and harmonic timbres shown in tables 2-1 and 2-2. Note that the lower completely non-harmonic component frequencies are shrunk, and become progressively less shrunk until they are stretched, and then the higher component frequencies become progressively more stretched.

Partial number (f_n)	**Completely non-harmonic Component frequency (Hz)**	**Ratio to f_1**	**Stretched Component frequency (Hz)**	**Ratio to f_1**	**Shrunk Component frequency (Hz)**	**Ratio to f_1**	**Harmonic Component frequency (Hz)**	**Ratio to f_1**
f_1	110	1	110	1	110	1	110	1
f_2	190.3	1.73	263.78	2.4	190.52	1.73	220	2
f_3	256.08	2.33	440	4	262.68	2.39	330	3
f_4	430.1	3.91	632.61	5.75	330	3	440	4
f_5	452.1	4.11	838.31	7.62	393.8	3.58	550	5
f_6	693	6.3	1055.12	9.59	455.07	4.14	660	6
f_7	738.1	6.71	1281.72	11.63	514.14	4.67	770	7
f_8	807.4	7.34	1516.9	13.79	571.56	5.2	880	8
f_9	1107.7	10.07	1760	16	627.55	5.71	990	9
f_{10}	1254	11.4	2010.25	18.28	682.11	6.2	1100	10
f_{11}	1531.2	13.92	2267.21	20.61	735.68	6.69	1210	11
f_{12}	1756.7	15.97	2530.33	23.00	788.15	7.17	1320	12
f_{13}	2006.4	18.24	2799.17	25.45	839.74	7.63	1430	13
f_{14}	2330.9	21.19	3073.62	27.94	890.56	8.1	1540	14
f_{15}	2989.8	27.18	3353.13	30.48	940.61	8.55	1650	15
f_{16}	3664.1	33.31	3637.7	33.07	990	9	1760	16

[45] Neville H. Fletcher and Thomas D. Rossing, *The Physics of Musical Instruments*, New York, Springer-Verlag, 1991, p. 73.

Study Number 1

Study Number 1 demonstrates the actual sound of harmonic, stretched and shrunk, and completely non-harmonic timbres. To some extent, it also demonstrates how timbre is constructed from the addition of pure tones by changing gradually from one timbre to another. To emphasise the focus on timbre, *Study Number 1* is texturally monophonic, and contains no harmony or melodic movement. Following is a list showing the order in which the timbres occur:

1. the harmonic timbre of an ideal (i.e., scientifically theoretical) plucked string
2. transition
3. the completely non-harmonic timbre of a struck circular membrane (such as that of a drum)
4. transition
5. a shrunk timbre (plucked)
6. transition
7. the completely non-harmonic timbre of a struck circular plate (such as that of a metal disc)
8. transition
9. a stretched timbre (plucked)
10. transition
11. the harmonic timbre of a plucked string

As the intention is only to demonstrate the sound of timbre in terms of it being a summation of pure tones, no effort has been made to emulate the sounds that occur in conjunction with acoustically produced timbres. That is, the sounds made by plectrums, percussion mallets, and sound-boards themselves, and most of the effects they can have on timbre have been ignored. The frequencies of the partials and their decay characteristics are the only way in which these timbres are designed to sound “natural”. The glissando effect apparent in the transitional sections is only due to the movement of the component frequencies; the frequency of the lowest and loudest partial is always fixed at 440 Hz (i.e., MIDI note number A 5).

How the component frequencies of a timbre can be known or predicted

Obviously, in order to derive a tuning system from the component frequencies of a timbre, the component frequencies must be known. The component frequencies of an acoustically produced timbre can be found via electronic analysis. This electronic analysis is often performed with a formula known as a “Fourier transform”. This thesis is concerned more with synthetic timbres, so a full description of the Fourier transform will not be given here. In essence though, a Fourier transform extracts the frequency, amplitude, and phase of each partial contained in a complex waveform (a waveform being a graph of a timbre that shows amplitude versus time) between two points in time. The small distance between the two points in time are kept at a constant, this constant being known as a “window”. A Fourier transform performed on many consecutive “windows” can show how the frequency, amplitude, and phase of each partial changes over the duration of the analysed timbre. A full description of the Fourier transform is given in *Elements of Computer Music*[46]. The phases of the partials in a timbre have not been mentioned so far because they have a negligible effect on audible timbre.

[46] F. Richard Moore, *Elements of Computer Music*, New Jersey, Prentice Hall, 1990, p. 62.

Filtration is another method of analysis that can be used for acoustically produced timbres. Again, because it pertains to acoustically produced timbres, only a brief and general description of this method is warranted here. Filtration analysis is performed by passing a signal (i.e., a timbre) through a set of parallel band-pass filters, in the same fashion as what is known as “phase vocoding”. Ideally, the band widths of these filters will not overlap, and will be narrow enough that only one partial of a timbre could pass through at a time; also, the centre frequencies would be such that the entire audible frequency spectrum is covered. As the centre frequency of each filter is known, and the amplitude of the output of each filter can be measured, the frequency and amplitude of each partial present in a timbre can be known.

There is little need for component frequency analysis of synthetic timbres, because most common synthesis methods allow the component frequencies of a timbre to be predetermined. In synthesizing timbre then, *predicting* component frequencies is more to the point than analysing them after the fact.

Amplitude modulation

In simple amplitude modulation synthesis, the output from one sine wave oscillator (the modulator) is summed with the amplitude of another sine wave oscillator (the carrier). Figure 2-1 shows this oscillator configuration.

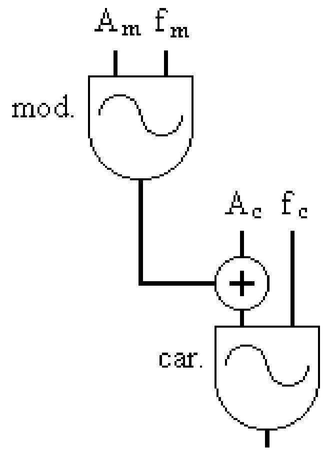

Figure 2-1. The oscillator configuration for the simple amplitude modulation of a sine tone, where A_m = the modulator's amplitude, f_m = the modulator's frequency, A_c = the carrier's amplitude, and f_c = the carrier's frequency.

With this configuration, the resultant timbre will consist of three component frequencies, equal to f_c , $f_c + f_m$ and $f_c - f_m$. The resulting partials (in this case $f_c + f_m$ and $f_c - f_m$) in amplitude, ring, or frequency modulation synthesis are known as "side-bands", because they occur on both sides (i.e., above and below) of the carrier frequency. Where the calculation of $f_c - f_m$ results in a negative frequency, this frequency is "reflected" around zero; for example, if f_c = 200 Hz and f_m = 500 Hz, the calculated frequency of the lower side-band will be –300 Hz. This side-band will be manifest as +300 Hz.

Digital synthesis presents its own difficulties, in that where the calculation of $f_c + f_m$ results in a frequency higher than the Nyquist frequency (which is equal to half of the sampling rate), the resulting frequency will be reflected around the Nyquist frequency, in the same way a negative frequency is reflected around zero. This phenomenon of reflected frequencies in digital synthesis is known as "aliasing" or "foldover".

The amplitude of the side bands will be equal to A_m / 2, while the amplitude of the "fundamental" (f_c) will be equal to A_c . The term "fundamental" is used tentatively here, as f_c may not necessarily be perceivable as the fundamental. If the frequency of a reflected partial happens to coincide with that of another partial, the amplitude of that partial may be affected, either constructively or destructively, depending on the amplitude and phase difference between the two partials.

If a complex waveform (a complex waveform being a waveform that contains partials, as opposed to a pure sine wave) is used as the modulator, the resulting component frequencies in the output will be f_c and $f_c \pm$ the frequency of each partial present in the modulator's waveform. The symbol ± here means + *and* −, not + *or* −. The amplitude of the side-bands corresponding to the most prominent partial in the modulator's waveform will be equal to $A_m / 2$, while the amplitudes of the rest of side-bands will be relative to this amplitude as their corresponding partials were to the most prominent frequency in the modulator's waveform. The amplitude of the "fundamental" (f_c) will be equal to A_c .

If a complex waveform is used as the carrier, but with a sine wave as the modulator, the resultant timbre will contain side-bands for each partial in the carrier's waveform. The relationship between each of the carrier's partials and their corresponding side-bands will be identical to the relationship explained above, between f_c and f_m , and A_c and A_m , when sine waves were used for both the modulator and the carrier. As would be expected, if complex waveforms are used for both the modulator and carrier, the resultant timbre could be very rich in partials, as it would contain all the partials present in the carrier's waveform, with each of these partials having side-bands for each partial present in the modulator's waveform.

Ring modulation

Ring modulation can be thought of as a form of amplitude modulation in which the output of the modulator is fed directly into the amplitude input of the carrier without being summed with another amplitude. Figure 2-2 shows this oscillator configuration.

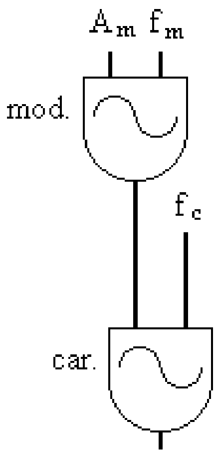

Figure 2-2. The oscillator configuration for the simple ring modulation of a sine tone, where A_m = the modulator's amplitude, f_m = the modulator's frequency, and f_c = the carrier's frequency.

The output of this oscillator configuration is identical to the output that would result from the multiplication of the two oscillators (see figure 2-3); assuming, of course, that the same oscillator frequencies were used. Therefore, ring modulation can also be thought of as the multiplication of two (or any number) of output signals. This also shows that it makes no difference as to which of the oscillators shown in figure 2-2 is used as the modulator or the carrier. The only difference there can be between the output of these two configurations is that, with the configuration shown in figure 2-3, the amplitude of the resultant timbre can be influenced by the second, and any other subsequent oscillators.

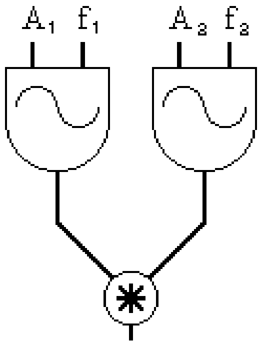

Figure 2-3. The output of this oscillator configuration would be identical to the output of the configuration shown in figure 2-2.

For the sake of simplicity, ring modulation will be explained in terms of the configuration shown in figure 2-2. If sine waves are used, there will be two component frequencies present in the resultant timbre, equal to $f_c + f_m$ and $f_c - f_m$. This result is identical to that gained from amplitude modulation, except that f_c is not present on its own. Again, where the calculation of $f_c - f_m$ results in a negative frequency, and where the calculation of $f_c + f_m$ results in a frequency higher than the Nyquist frequency (in digital synthesis), these frequencies are “reflected”. The amplitudes of the partials present in the resultant timbre will be equal to $A_m / 2$, identical again to the result gained from the amplitude modulation.

If a complex waveform is used for the modulator but not the carrier, or for the carrier but not the modulator, or for the carrier *and* the modulator, the results will all be as they would for amplitude modulation, but without the presence of any of the carrier’s component frequencies. It is possible though, that a side-band in the resultant timbre could be identical to a component frequency that was present in the carrier’s waveform.

Frequency modulation

In simple frequency modulation synthesis, the output from one sine wave oscillator (the modulator) is summed with the frequency of another sine wave oscillator (the carrier). Figure 2-4 shows this oscillator configuration.

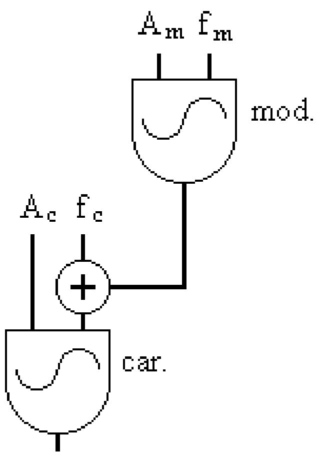

Figure 2-4. The oscillator configuration for the simple frequency modulation of a sine tone, where A_m = the modulator's amplitude, f_m = the modulator's frequency, A_c = the carrier's amplitude, and f_c = the carrier's frequency.

With this configuration, the resultant timbre can, theoretically, consist of an infinite number of partials. The component frequencies present in the resultant timbre of the oscillator configuration shown above will be equal to f_c , $f_c \pm f_m$, $f_c \pm 2(f_m)$, $f_c \pm 3(f_m)$, $f_c \pm 4(f_m)$ and so on, such that the value of any component frequency is $f_c \pm n(f_m)$. The symbol ± here means + *and* –, not + *or* –. It can be seen that the frequencies of the first side-band pair occur as they would with amplitude modulation, and that each subsequent partial is an addition or subtraction of a multiple of f_m. For explanation's sake, these subsequent partials will be referred to as "side-band multiples". Again, where the calculation of $f_c - n(f_m)$ results in a negative frequency, and where the calculation of $f_c + n(f_m)$ results in a frequency higher than the Nyquist frequency (in digital synthesis), these frequencies are "reflected".

The number of partials present in the resultant timbre, and their amplitude relative to A_c , is controlled by the value of A_m. Exactly how the value of A_m affects the number of partials and their relative amplitudes can be determined by formulae known as "Bessel functions". Bessel functions are too complex to delve into here, but a full description is given in *Elements of Computer Music*[47]. It can generally be said, though, that the higher the value of A_m , the greater the number of side-bands there will be in the resultant timbre. It can also be said, generally, that the amplitude of each subsequent side-band pair is less than the previous one; that is, the side-bands at the extremes of the frequency spectrum of a timbre created through frequency modulation have low amplitudes, while the side-bands at, or near, the middle of the frequency spectrum of the timbre have high amplitudes.

If a complex waveform is used as the modulator, the results will be as they would for amplitude modulation, and with side-band multiples of each component frequency present in the modulator's waveform. If a complex waveform is used as the carrier, the results will be as they would for amplitude modulation, and with side-band multiples of the modulator frequency present for each component frequency present in the carrier's waveform. As would be expected, if complex waveforms are used for both the modulator and carrier, the resultant timbre would contain side-band multiples of each component frequency in the modulator's waveform for each component frequency in the carrier's waveform. This can result in a timbre so rich in partials that it is virtually white noise.

Additive synthesis

For additive synthesis, the output from numerous sine wave oscillators are summed. Figure 2-5 shows this oscillator configuration.

[47] F. Richard Moore, *Elements of Computer Music*, New Jersey, Prentice Hall, 1990, p. 320.

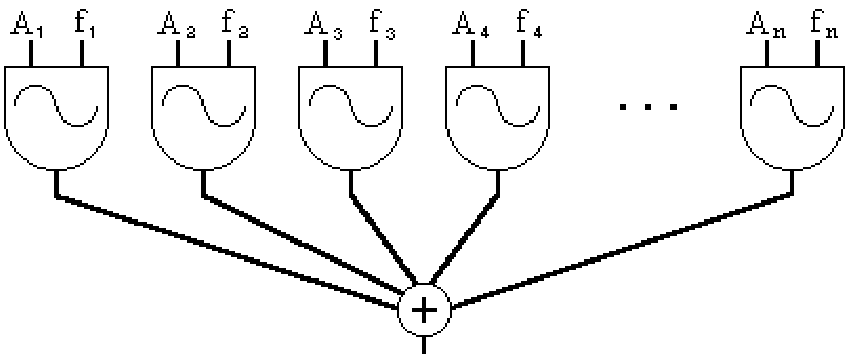

Figure 2-5. The oscillator configuration for additive synthesis, where A = the amplitude of the corresponding oscillator, and f = the frequency of the corresponding oscillator.

The frequency and amplitude of each partial present in the resultant timbre will be known because they are directly specified by the "synthesist". While the synthesis methods explained previously allow the creation of, and transition between, harmonic, stretched and shrunk, and completely non-harmonic timbres, additive synthesis allows all this, plus the ability to extensively change the relationships between any partial and any other partial (or partials). For example, in amplitude, ring, and frequency modulation synthesis, the relationship between the partials of the resultant timbre and the controllable parameters (i.e., the amplitudes and frequencies of the modulator and the carrier) remain fixed, regardless of what the values of the controllable parameters are, while with additive synthesis, this is not the case.

Although additive synthesis presents a tedious process in the production of partial rich timbres (especially when it is considered that each oscillator should, ideally, have its own envelope), it is particularly useful for the purposes of this thesis, because the tuning systems that can be derived from additively synthesised timbres will not be restricted to reflections of the fixed relationships apparent between the partials of the resultant timbre, and the controllable parameters of amplitude, ring, and frequency modulation. Additive synthesis also allows many more possibilities for the production of timbre to suit existing or pre-conceived alternate tuning systems.

Deriving a timbre from a given tuning system

For this thesis, the purpose of deriving the component frequencies of a timbre from a given (i.e., existing or pre-determined) tuning system will be to achieve acoustically consonant intervals and chords. The first step in doing this would be to decide which intervals and chords of the tuning system are desired to be acoustically consonant.

In tuning systems intended for use with harmonic timbres, such as meantone temperament, 12 tone equal temperament or just intonation, the intervals of the octave, fifth, and major third result in the highest degree of acoustical consonance when sounded with harmonic timbres. This is because these intervals occur low in the harmonic series, and are therefore often highest in amplitude of the partials comprised in a typically harmonic timbre. This means that they are most significant in the determination of which intervals and chords sound acoustically consonant.

If it were desired that the intervals of the major seventh and the tritone in 12 tone equal temperament be acoustically consonant, it could be achieved with a timbre comprised of high amplitude component frequencies at these intervals to each other. The calculation of these component frequencies relative to the fundamental would simply be the same as calculating the pitch frequencies relative to a reference tone (the reference tone being the pitch frequency from which all others can be calculated) in 12 tone equal temperament.

Table 2-4 shows possible component frequencies (with A at 110 Hz as the fundamental) of a timbre that could provide acoustically consonant major sevenths and tritones in 12 tone equal temperament. It also shows the equivalent pitch (in 12 tone equal temperament) of each component frequency, and that the intervals between them are often major sevenths and tritones. For the timbre to be most convincing as one that could be imagined as being produced acoustically, the difference between the component frequencies decreases as they get higher, as happens in the harmonic series. Table 2-5 shows the same component frequencies as table 2-4, but relative to G# (at 207.65 Hz). This illustrates that there are a number of component frequencies in common between these identical timbres when their fundamentals (f_1) are an equally tempered major seventh apart. Table 2-6 shows that there are component frequencies in common when the fundamentals are an equally tempered tritone apart.

Table 2-4. The component frequencies (relative to A at 110 Hz) that could possibly constitute a timbre that can provide acoustically consonant major sevenths and tritones in 12 tone equal temperament, and the pitches that correspond to them.

Partial number (f_n)	**Component frequency (Hz)**	**Corresponding pitch (in 12 tone equal temperament, expressed as a MIDI note)**
f_1	110	A 2 major 7th
f_2	207.65	G# 3 major 7th tritone
f_3	293.66	D 4
f_4	392	G 4 tritone
f_5	466.16	A# 4 major 7th
f_6	554.37	C# 5
f_7	739.99	F# 5

Table 2-5. The same component frequencies (relative to the fundamental) as table 2-4, but an equally tempered major seventh higher. Note that there are several frequencies (circled) that coincide exactly with those of table 2-4, meaning that the interval of an equally tempered major seventh will result in high acoustical consonance.

Partial number (f_n)	**Component frequency (Hz)**	**Corresponding pitch (in 12 tone equal temperament, expressed as a MIDI note)**
f_1	207.65	G# 3
f_2	392	G 4
f_3	554.37	C# 5
f_4	739.99	F# 5
f_5	880	A 5
f_6	1046.5	C 6
f_7	1396.91	F 6

Table 2-6. The same component frequencies again (relative to the fundamental) as table 2-4, but this time an equally tempered tritone higher. Here there are only two frequencies (circled) that coincide with those of table 2-4. This means that although the interval of an equally tempered tritone will result in a degree of acoustical consonance, it will not be as acoustically consonant as an equally tempered major seventh. This parallels the fact that a justly intoned fifth sounded by a harmonic timbre is not as acoustically consonant as an octave, although it is still acoustically consonant.

Partial number (f_n)	**Component frequency (Hz)**	**Corresponding pitch (in 12 tone equal temperament, expressed as a MIDI note)**
f_1	155.56	D# 3
f_2	293.66	D 4
f_3	415.3	G# 4
f_4	554.37	C# 5
f_5	659.26	E 5
f_6	783.99	G 5
f_7	1046.5	C 6

Through observation of the intervals between these component frequencies, and of the component frequencies in common when the fundamentals of timbres comprised of these component frequencies are at particular intervals, it can be noticed that equally tempered fourths and minor thirds also result in acoustical consonance with this timbre.

Chapter 3: Tuning

The "harmonic scale"

The harmonic scale is a tuning system that is an extended form of just intonation. The early part of this thesis explained how the pitch frequencies of just intonation are calculated from the frequency ratios apparent between the intervals of the major and minor third as they appear in the harmonic series. These frequency ratios are 5:4 and 6:5 respectively. By using these frequency ratios in the calculation of just intonation, the frequency ratios apparent between the major second, perfect fifth, major seventh, and octave (relative to the reference tone, in this case A at 440 Hz) are also as they appear in the harmonic series. These frequency ratios are 9:8, 3:2, 15:8, and 2:1 respectively. It can therefore be said that a considerable number of justly intoned intervals or chords sounded with harmonic timbres result in a high degree of acoustical consonance.

The harmonic scale contains 12 chromatic pitches, the frequency ratio of every pitch relative to the tonic (or reference tone, which for most forms of just intonation can be considered as the tonic) being as it appears in the harmonic series. As such, any interval greater than or equal to an approximate minor third (i.e., the critical bandwidth), or chord in which no two pitches are voiced with less than an approximate minor third between them, sounded with harmonic timbres will produce complete, or at least nearly complete, acoustical consonance.

Of course, it is inevitable that some harmonic timbres will contain partials that are within the critical bandwidth of each other. It is with these timbres that only nearly complete acoustical consonance can be attained. This is because roughness occurs as part of the timbre itself, when it is not part of an interval or chord. However, the harmonic partials comprised in a timbre that can be within the critical bandwidth of each other can't occur until at least the seventh harmonic partial (counting the fundamental as the first), and the higher partials in a timbre are generally quite low in amplitude in relation to the lower partials, so usually cause only minimal beating.

Because all frequency ratios in the harmonic scale are taken directly from the harmonic series, and many timbres are comprised of partials of the harmonic series of their fundamental tone, the harmonic scale is a good example of a tuning system derived from the component frequencies of timbre. Appendix VI shows more detail of the harmonic scale.

Carlos[48] constructed a 144 tone per octave (non-equally tempered) tuning by calculating a new harmonic scale from each pitch of an existing (i.e., pre-calculated) harmonic scale. This 144 tone tuning allows modulation around a full cycle of key signatures while keeping the frequency ratios of all pitches relative to the tonic identical for each key, and has been implemented in piece titled *Just Imaginings*[49].

Sethares's dissonance curves

Instead of taking the frequency ratios for a tuning system directly from the ratios between the partials of a timbre, Sethares[50] finds the frequency ratios for a timbre derived tuning system from what he terms as a "dissonance curve". A dissonance curve is a graph depicting the level of acoustical dissonance against the frequency ratio of intervals sounded

[48] Wendy Carlos, "Tuning: At The Crossroads", *Computer Music Journal*, Vol. 11, no. 1, 1987, p. 29-43.

[49] Wendy Carlos, "Just Imaginings", *Beauty in the Beast*, Audion (Jem), SYNCD200, 1987.

[50] William A. Sethares, "Local Consonance and the Relationship Between Timbre and Scale", *Journal of the Acoustical Society of America*, Vol. 94, no. 3, 1993, p. 1218-1228, and William A. Sethares, "Relating Timbre and Tuning", *Experimental Musical Instruments*, Vol. 9, no. 2, 1993, p. 22-29.

with any given timbre. The minima of the graph show the intervallic frequency ratios that will provide the highest degree of acoustical consonance for the given timbre. These frequency ratios can be used to calculate the frequencies of a tuning structure that ideally (in terms of acoustical consonance) suits the given timbre. Figure 3-1 shows an example of a dissonance curve.

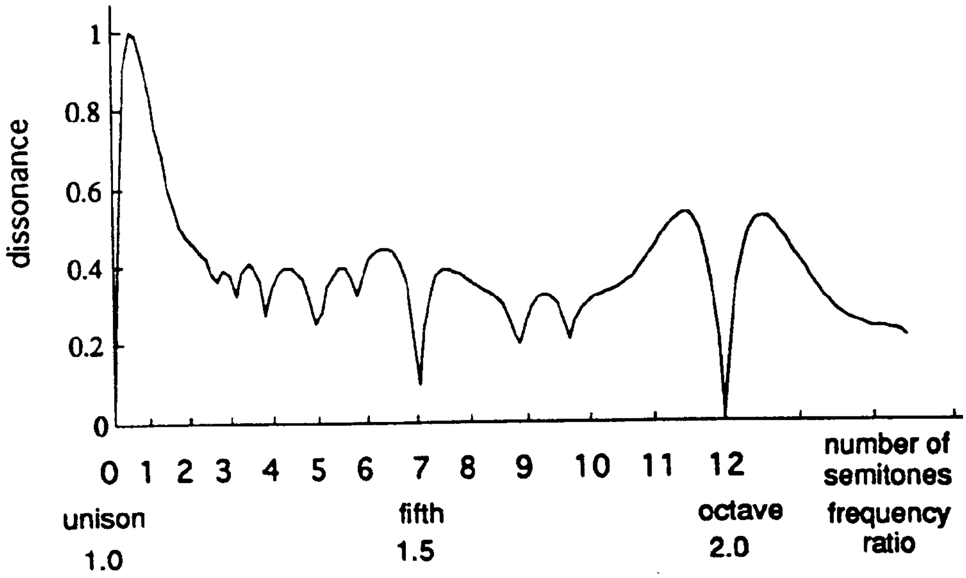

Figure 3-1. A diagram by William Sethares[51]. The dissonance curve for a typical harmonic timbre. Most minima occur at justly intoned frequency ratios. The hump near the vertical axis depicts the critical bandwidth.

The coordinates of a dissonance curve are calculated by a computer program. The number of partials, and the frequency and amplitude of each partial in a timbre are input to the program by the user. Taking into account the amplitude and frequency of each partial, the program assesses the acoustical consonance of intervallic frequency ratios for the timbre from 1:1 (i.e., unison) to 2.2:1, at increments of 0.01. For example, the acoustical consonance of the frequency ratio 1:1 is assessed, then 1.01:1, 1.02:1, 1.03:1, and so on until 2.2:1. The starting and ending ratios can be reset by the user, as can the

[51] William A. Sethares, "Local Consonance and the Relationship Between Timbre and Scale", *Journal of the Acoustical Society of America*, Vol. 94, no. 3, 1993, p. 1218-1228, and William A. Sethares, "Relating Timbre and Tuning", *Experimental Musical Instruments*, Vol. 9, no. 2, 1993, p. 22-29.

incrementation, so for greater accuracy over a larger span of frequency ratios, the program could, for instance, assess the acoustical consonance of intervallic frequency ratios from 1:1 to 4:1, at increments of 0.001. Appendix VII shows Sethares's computer program for calculating the coordinates of a dissonance curve.

While building a tuning structure from the frequency ratios where the minimum points of a dissonance curve occur for a given timbre would often yield identical results to deriving a tuning structure directly from the frequency ratios between the partials of that timbre, use of the dissonance curve has four major advantages:

1. The calculation of the curve takes into account the amplitude of each partial present in a timbre. By doing this, partials that have amplitudes not significant enough to cause sufficiently audible beating are barely taken into account.

2. The dissonance curve shows that there are degrees of acoustical consonance, breaking the assumption that an intervallic frequency ratio for any given timbre is either consonant or dissonant, and that consonance and dissonance are mutually exclusive states. For example, the dissonance curve for a harmonic timbre (that contains the first seven partials of the harmonic series of its fundamental) has minima at the frequency ratios 4:3 and 3:2, but the minimum at 4:3 is considerably less consonant than the one at 3:2.

3. The dissonance curve not only shows minima at intervallic frequency ratios where there are component frequencies common to both pitches (which are the frequency ratios that can be taken directly from the ratios between the component frequencies of the timbre), but also at points where the component

frequencies of both pitches of the interval are all sufficiently separated for beating not to occur. This makes acoustically consonant frequency ratios available that would have been overlooked when taking frequency ratios directly from the ratios between the partials of the given timbre.

4. The calculation of the dissonance curve recognises that a timbre itself can be intrinsically dissonant by containing high amplitude partials that have frequencies within approximately a minor third (i.e., the critical bandwidth) of each other, causing "roughness" even at unison or as a solo tone. The dissonance curve of such a timbre would comprise no minimum points. If there is roughness intrinsic to a timbre itself, taking the frequency ratios for a tuning system directly from the ratios between the partials will only yield *least dissonant* (as opposed to most consonant) intervals. That is, the intervallic frequency ratios gained in this manner could only serve to minimise the beating between partials, as beating will always occur regardless. This is not to say that a tuning system derived in this manner for such a timbre may not be useful.

Slaymaker's stretched scales

Working with the assumption (or acceptance) that 12 tone equal temperament provides close enough approximations of justly intoned intervals to provide acoustically consonant intervals and chords for timbres comprised of harmonic partials, Slaymaker[52] explored the potential of stretching or shrinking 12 tone equal temperament in order to suit (in terms of acoustical consonance) stretched or shrunk timbres. Reiterating equation 2, the

[52] Frank H. Slaymaker, "Chords from Tones Having Stretched Partials", *Journal of the Acoustical Society of America*, Vol. 47, 1970, p. 1569-1571.

component frequencies of stretched or shrunk timbres can be calculated with the expression:

let f_n = the frequency of the nth partial

S = index of stretching or shrinkage (equal to 1 for the harmonic series)

$$f_n = n^S(f_1)$$

The frequency ratio of each pitch in 12 tone equal temperament to its next lowest pitch is $2^{1/12}$, or ($\sqrt[12]{2}$). For a stretched or shrunk timbre, where *S* is the index of stretching or shrinkage, the frequency of each pitch in a tuning system that approximates acoustically consonant intervals and chords can be expressed as a ratio of $(2^{1/12})^S$ to the next lowest pitch. This achieves a 12 tone equal division of a stretched or shrunk octave.

While this idea of a stretched scale is not strictly a tuning system derived from the component frequencies of a timbre, the component frequencies of timbre have been taken into considerable account. Of course, a 12 tone equally tempered division of a stretched or shrunk octave is not likely to provide any better approximation of acoustically consonant intervals or chords for stretched or shrunk timbres than conventional 12 tone equal temperament provides for harmonic timbres, but this method of stretching or shrinking scales could simply be applied to a greater number of equal divisions of the octave, such as 19, 31, or 53, in order to provide exact, or at least very close approximations to acoustically consonant intervals or chords for stretched or shrunk timbres.

Study Numbers 2a, 2b and 2c

These studies are intended to demonstrate the audible effect of tuning systems derived from the component frequencies of timbre. Tuning and timbre constitute the only differences between these studies; otherwise, in terms of form, texture, tempo, rhythm, and melodic and harmonic structure, each is identical. This emphasises the focus on tuning as it is related to timbre, and allows an accurate comparison of the effects apparent in each study. An arpeggiated chord forms the basis of these studies. As each note remains sounding until its next attack, the combined effects and tuning on timbreare conveyed in both harmonic and melodic terms.

In *Study Number 2a*, the timbre moves as it does in *Study Number 1*. The tuning of the arpeggiated chord remains fixed to the first seven partials of the harmonic series of the first note, which is the root and bass of the chord. The tuning is only related to the timbre when the timbre is harmonic.

In *Study Number 2b*, the timbre moves as it does in *Study Number 1*, and the tuning of the arpeggiated chord changes to match the first seven partials of each timbre. That is, the tuning corresponds directly to the timbre in each timbrally stable section.

In *Study Number 2c*, the timbre remains fixed as the harmonic timbre of an ideal plucked string, and the tuning moves as it does in *Study Number 2b*. That is, the tuning is only related to the timbre when the *tuning* is harmonic.

Due to the somewhat dense nature of the rhythmic and harmonic texture of these studies, acoustical dissonance can not really be perceived as *beating*. This is because there can be so many rates of beating occurring simultaneously that acoustical dissonance is more likely to be perceived as some form of cacophony. This has no impact on the demonstrative qualities of these studies, as their intention is to show the audible effects of timbre related tunings in a *musical* context. For this reason acoustical consonance and dissonance will be discussed here in a fairly general context.

From a subjective viewpoint, *Study Number 2a*, on its first hearing, probably sounds quite alright as far as consonance is concerned. This would probably be because the tuning is close to those to which most people are accustomed. *Study Number 2b*, despite containing tunings that are most likely not that to which many people are accustomed, probably tends to sound quite alright too, as far as consonance is concerned. *Study Number 2a*, on its second hearing, after listening to *Study Number 2b*, may tend to sound cacophonous. *Study Number 2b*, on its second hearing, after hearing *Study Number 2a* as cacophonous, probably tends to sound particularly “clean” and clear.

Study Number 2c, utilising tunings that are most likely not those to which many people are accustomed with a harmonic timbre might sound cacophonous, but probably just tends to sound “out of tune”. This is despite the fact that these tunings sounded fine in *Study Number 2b*. These observations attest strongly to the derivation of tuning systems from the component frequencies of timbre for the purpose of acoustical consonance.

Addressing the unaddressed issues

The review of research literature in the field of timbre derived tuning systems revealed some issues which are yet to have been tackled in much depth. The first of these was the concept of a tuning system that is designed to *minimise* the acoustical consonance of intervals and chords sounded by particular timbres. This would obviously be for the purpose of maximising acoustical dissonance. Such a tuning system could be used in conjunction with one that provides maximum acoustical consonance. By alternating between the two systems through a passage of harmony, the audible interaction between tension and resolution would theoretically be improved. The most efficient method of deriving a tuning system from the partials of a timbre for the purpose of achieving acoustical dissonance would be to use frequency ratios occurring at the maximum (instead of the minimum) points of a dissonance curve[53].

The second of these untackled issues is the dilemma of developing a tuning system that provides acoustically consonant intervals and chords for a harmonic and non-harmonic timbre sounding simultaneously. One way to go about developing such a tuning system might be to use frequency ratios that are apparent between the partials of both of the concerned timbres. If there were to be any component frequency ratios common to both timbres, it would only be a few, so this method, if feasible for the timbres concerned, would probably only result in a system with severely restricted pitch possibilities. Another way to go about developing such a tuning system could be to interpolate between the minimum points of each timbre's dissonance curve[54], but this would most likely result in a tuning system that doesn't work with either of the timbres concerned.

The unlikelihood of a tuning system providing completely acoustically consonant intervals and chords for a harmonic and non-harmonic timbre sounding simultaneously means that some compromise is necessary. This compromise would best be achieved by "tempering" (i.e., adjusting) the pitch frequencies of the most acoustically consonant tuning system of one timbre as little as possible, such that they are as closely in tune as possible to the most acoustically consonant tuning system of the other timbre. An example of this practice is given by Carlos[55], who re-tuned the pitch frequencies of a scale suited to the

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[53] William A. Sethares, "Local Consonance and the Relationship Between Timbre and Scale", *Journal of the Acoustical Society of America*, Vol. 94, no. 3, 1993, p. 1218-1228, and William A. Sethares, "Relating Timbre and Tuning", *Experimental Musical Instruments*, Vol. 9, no. 2, 1993, p. 22-29.

[54] William A. Sethares, "Local Consonance and the Relationship Between Timbre and Scale", *Journal of the Acoustical Society of America*, Vol. 94, no. 3, 1993, p. 1218-1228, and William A. Sethares, "Relating Timbre and Tuning", *Experimental Musical Instruments*, Vol. 9, no. 2, 1993, p. 22-29.

[55] Wendy Carlos, "Tuning: At The Crossroads", *Computer Music Journal*, Vol. 11, no. 1, 1987, p. 36.

gamelan to be as acoustically consonant as possible with the pitch frequencies of just intonation.

Electronically realising a timbre derived tuning system

The MIDI tuning standard

In 1991, Carter Scholz[56] proposed an extension to the MIDI specification to the IMA (International MIDI Association). This extension to the MIDI specification was intended as standard format by which MIDI compatible devices could share tuning system data, for the purpose of making alternate tuning systems more available to users of MIDI devices.

The proposal suggested three main methods by which a peripheral controller (such as a master keyboard or sequencer) could specify or alter a device's tuning data:

1. In the form of system exclusive data, a "tuning program" (analogous to a MIDI program number, but referring to a tuning system) and its comprised data could be transmitted or received by a device in groups of 3 bytes at a time for each MIDI note number in turn (for example, MIDI note number 60 = middle C). The first byte specifies the MIDI note number to be tuned, and the following two bytes specify the deviation of that note number in cents (a cent being 1/100 of a 12 tone equally tempered semitone, or 1/1200 of an octave) from its value in 12 tone equal temperament. When the transmission (and reception) is complete,

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[56] Carter Scholz, "A Proposed Extension to the MIDI Specification Concerning Tuning", *Computer Music Journal*, Vol. 15, no. 1, 1991, p. 49-54.

each MIDI note number of the receiving device will produce the corresponding pitch frequency specified in the communicated tuning system. This method of tuning system transfer would not occur in real-time (i.e., the data transmission is too slow not to disrupt a performance).

2. Also in the form of system exclusive data, the tuning of any one particular note could be altered within a device's current tuning program. Because only one MIDI note number is affected, the amount of data communicated is small, making the transmission faster and thereby allowing this method to be used in real-time.

3. Rather than communicating actual tuning data via system exclusive messages, Scholz's proposal[57] suggests the allotment of MIDI controller number 3 for the selection of tuning programs stored within a device, in much the same way as a MIDI "program change" specifies a sound to be used from a device's internal memory.

While the proposed MIDI tuning standard provides a means of sharing high resolution tuning data between devices, altering a device's tuning from a peripheral controller, and selecting a device's tuning program from a peripheral controller, it has three main deficiencies:

[57] Carter Scholz, "A Proposed Extension to the MIDI Specification Concerning Tuning", *Computer Music Journal*, Vol. 15, no. 1, 1991, p. 49-54.

1. The tuning of a MIDI note number cannot be less than the value of its frequency in 12 tone equal temperament (i.e., pitches can only be tuned up, not down).

2. The tuning of a MIDI note number is adjustable to only 100 cents (i.e., one equally tempered semitone) above the value of its frequency in 12 tone equal temperament.

3. While the proposed standard allows tuning to be altered during performance, and allows different devices to be in different tunings simultaneously, the tuning for any one device is global. On a single “multi-timbral” device, this means, for instance, that it is impossible for the MIDI program on channel 1 to be simultaneously in a different tuning to the MIDI program on channel 2.

MIDI controllable synthesizers, or samplers, that don’t conform to the MIDI tuning standard

Tuning systems other than 12 tone equal temperament can be achieved in a variety of ways with many MIDI controllable synthesizers and samplers that don’t conform to the MIDI tuning standard.

One of these ways is the “re-scaling” of pitch across the range of the keyboard. This is analogous to the way in which parameters such as amplitude, envelope times, and filter cut-off frequencies can be scaled to respond at a level determined by MIDI note numbers. The re-scaling of pitch across the keyboard can be useful for achieving equal divisions of any particular interval, and therefore is ideal for the realisation of tuning systems such as (for example) quarter-tone (i.e., 24 tone equal) temperament, 19, 31, or 53 tone equal temperaments (for their close approximations to justly intoned intervals), or equal divisions of stretched or shrunk octaves. The drawback of using this method for large number divisions of the octave (for instance, 53 tone equal temperament) is that the MIDI specification allows for only 128 note numbers, and most MIDI controllable devices only respond to 88 of these, meaning that for a tuning system such as 53 tone equal temperament, it is likely that less than two octaves of pitches will be available in any one MIDI program.

Another way that tuning systems can be realised on many MIDI controllable synthesizers and samplers is the tuning of notes in octaves. An amount of deviation from 12 tone equal temperament (usually in cents) can be specified for each pitch within an octave. The pitches in every octave are then affected by the same specified deviation. For example, tuning C to be 50 cents sharp will make C 50 cents sharp in every octave. Using this method is useful for achieving any tuning system that contains 12 pitches per keyboard octave, and therefore is ideal for the realisation of meantone temperaments and many forms of just intonation.

Another, and more versatile, way that some MIDI controllable synthesizers and samplers can be configured to realise tuning systems other than 12 tone equal temperament is by the tuning of each individual note number. The only drawback of this method is that the tuning resolution is often not very high; that is, each MIDI note number may only be capable of being tuned in increments of 1.6 cents. If the desired frequency for a particular note number lies between the increments, and must therefore be approximated, this inaccuracy can be quite audible.

If controlling a synthesizer or sampler from a sequencer, a calculated pitch bend value can be sent before each note to adjust its 12 tone equal tempered frequency to the one that is desired. This method has three main drawbacks:

1. Pitch bend data affects *all* of the pitches on the same MIDI channel to the same extent, making it unlikely that the pitches of an interval or chord can be of their desired frequencies unless each pitch is assigned to its own channel, so each MIDI channel will most likely need to be texturally monophonic for this method to be successful.

2. Even if the note data on each MIDI channel is kept texturally monophonic, pitch bend occurs virtually instantly, so pitch bend data of any subsequent note can quite possibly affect the end of the previous note that is still audible due to its envelope's release segment.

3. While the MIDI specification allows for 14 bit resolution of pitch bend data, many synthesizers and samplers only support 7 bit resolution which, if the pitch bend range is set to up or down one semitone, works out to tuning increments of only 64 steps per semitone, or 1.6 cents. As described above, this is barely accurate enough.

With some MIDI controllable synthesizers, and most samplers, it is possible to tune the sample that has been assigned to a span of note numbers. If a sample is duplicated for however many pitches are desired for a tuning system, and each duplicate of the sample is assigned to only one note number, it will therefore be possible to tune each note number individually. This can be done to the accuracy of whatever is the sample tuning resolution of the device, which is usually 1 cent. This method, although laborious, has few of the drawbacks of any of the methods described above, and shares the capabilities of all of these methods. It may also be used in conjunction with any of the methods described above, depending entirely on the particular device being used. As a sampler can play-back the timbre from which a tuning system may have been derived, and can play-back any timbre that suits an existing or pre-conceived alternate tuning system, this method is particularly useful for the purposes of this thesis.

Direct Digital Synthesis (Csound)

While a MIDI controlled sampler presents a practical, efficient, and fairly versatile means of realising music that utilises timbre derived tuning systems, or tuning system derived timbres, a MIDI controlled sampler is deficient in that it can't do much more with a specific timbre than simply play it back at a specified frequency. This thesis is concerned with tuning systems that are directly related to the component frequencies of timbre, so if a conceived piece of music requires a gradual change in the component frequencies of a timbre, it could also require a gradual change from one tuning system to another. A MIDI controlled sampler may be capable of executing a gradual change from one tuning system to another, but is incapable (at this point in time) of executing a gradual change in the component frequencies of a timbre as it is playing it back. Direct digital synthesis is capable of this.

A software synthesis program called Csound, written by Barry Vercoe at the Massachusetts Institute of Technology, is currently the most prevalent technique for performing the direct digital synthesis of music. Csound can realise any tuning system in numerous ways, such as directly specifying the desired frequency of each pitch, specifying a formula for the calculation of the frequency of each pitch, defining a table to which a Csound instrument can refer to find the frequency of each pitch, or specifying the deviation from a frequency in 12 tone equal temperament for each pitch to be played.

Csound is also capable of many synthesis techniques (including all that were described in chapter 2), and extensive timbral manipulation of sampled sounds. This makes Csound a highly feasible method for the realisation of music in which tuning systems can gradually change to stay in direct relation to the gradual changing of the component frequencies of timbre, or music in which the component frequencies of timbre can gradually change to stay in direct relation to the gradual changing of a tuning system. Appendix V shows the Csound instrument used to create the studies referred to in this thesis.

Conclusion

This thesis began by describing the functions, deficiencies, and calculation of pitch frequencies in the main Western tuning systems. This provided an introduction to the topic, and illuminated the fact that the 12 tone equal temperament to which the modern Western world is accustomed does not provide optimum results in situations where the acoustical consonance of intervals and chords is of paramount importance. After looking at the idea of acoustical consonance, and the importance of this to the development of tuning systems, the concept of timbre was discussed.

It was explained how timbre is constructed from multiple pure tones called partials (or component frequencies). The different relationships between the partials comprised within timbre led to the categorisation of timbre into the divisions of harmonic, stretched and shrunk, and completely non-harmonic. An explanation of amplitude modulation, ring modulation, frequency modulation and additive synthesis then showed how timbres in each of these categories can be synthetically produced. It was then shown how the partials of a timbre can be derived from a given tuning system for the purpose of making particular intervals and chords sound acoustically consonant.

The idea of tuning systems that are directly related to the partials of timbre for the purpose of achieving acoustically consonant intervals and chords was then discussed from the perspectives of other writers in the field, namely Carlos[58], Sethares[59] and Slaymaker[60].

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[58] Wendy Carlos, “Tuning: At The Crossroads”, *Computer Music Journal*, Vol. 11, no. 1, 1987, p. 29-43.

[59] William A. Sethares, “Local Consonance and the Relationship Between Timbre and Scale”, *Journal of the Acoustical Society of America*, Vol. 94, no. 3, 1993, p. 1218-1228, and William A. Sethares, “Relating Timbre and Tuning”, *Experimental Musical Instruments*, Vol. 9, no. 2, 1993, p. 22-29.

A series of musical studies was then presented to demonstrate the audible effect of timbre derived tuning systems.

Following this was a discussion of some issues related to timbre derived tuning systems that have been given little or no attention in the examined research literature. The result of this was a look into some previously unexplored territory, with some doors being opened for further investigation. Finally, it was explained how music in which tuning relates to timbre can be realised in electronic music, with either a MIDI system or direct digital synthesis. This was an important area to explore, as the theory of timbre related tuning is useless if it can not be practiced.

The most substantial accomplishment of this paper would probably be in the fact that it has gone beyond the mere discussion of the topic's theory, and presented its findings in an audible and musical context. *Study Number 2b* demonstrated the effect of acoustical consonance achieved by the direct correspondence of a chord's tuning to a timbre's component frequencies. More importantly though, it showed that timbre related tunings can provide musically useful possibilities that might otherwise be overlooked.

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[60] Frank H. Slaymaker, "Chords from Tones Having Stretched Partials", *Journal of the Acoustical Society of America*, Vol. 47, 1970, p. 1569-1571.

Appendices

I. Pythagorean tuning

MIDI note	Equation	Alternative equation	Frequency (Hz) (to 2 decimal places)	Cents (from A4)	Cents (from previous note)
A 4	n/a	n/a	440	0	n/a
A# 4	$((3f_{D\#5}) / 2) / 2$	$(3f_{D\#5}) / 4$	469.86	114	114
B 4	$((3f_{E5}) / 2) / 2$	$(3f_{E5}) / 4$	495	204	90
C 5	$((3f_{F5}) / 2) / 2$	$(3f_{F5}) / 4$	528.6	318	114
C# 5	$((3f_{F\#5}) / 2) / 2$	$(3f_{F\#5}) / 4$	556.88	408	90
D 5	$((3f_{G5}) / 2) / 2$	$(3f_{G5}) / 4$	594.67	521	114
D# 5	$((3f_{G\#5}) / 2) / 2$	$(3f_{G\#5}) / 4$	626.48	612	90
E 5	$(3f_{A4}) / 2$	$2((3f_{A4}) / 4)$	660	702	90
F 5	$(3f_{A\#4}) / 2$	$2((3f_{A\#4}) / 4)$	704.79	816	114
F# 5	$(3f_{B4}) / 2$	$2((3f_{B4}) / 4)$	742.5	906	90
G 5	$(3f_{C5}) / 2$	$2((3f_{C5}) / 4)$	792.89	1019	114
G# 5	$(3f_{C\#5}) / 2$	$2((3f_{C\#5}) / 4)$	835.31	1110	90
A 5	$2f_{A4}$	n/a	880	1200	90

Pythagorean tuning can be correctly calculated with the ratio of the justly intoned 4th (i.e., 4:3) instead of the justly intoned 5th (i.e., 3:2) because the ratio 4:3 gives the intervallic inversion of the justly intoned 5th. The frequency ratio that gives the inversion of a justly intoned interval can be found by doubling the denominator of the un-inverted frequency ratio (which in the case of 3:2, is 2), and then reciprocating the ratio (i.e., turn it upside down); so what was 3:2, becomes 3:4, and then 4:3 (which is essentially the same as 3:4, differing only in that it gives an ascending interval).

It will be noticed that $f_{A5} = 2f_{A4}$, and not $(3f_{D5}) / 2$. This is because there is a discrepancy between the frequency that can be calculated as a just octave from the reference tone (or tonic, in this case A4), and as a just fifth from the fourth of the scale (in this case D5). The ratio of the frequency difference of this discrepancy is known as the "Pythagorean comma"[61], and is 531441:524288.

[61] James Murray Barbour, *Tuning and Temperament A Historical Survey*, New York, Da Capo Press, 1972, p. V.

II. 1/4 comma meantone temperament

MIDI note	Equation	Frequency (Hz) (to 2 decimal places)	Cents (from A4)	Cents (from previous note)
A 4	n/a	440	0	n/a
A# 4	$((3f_{D\#5}) / 4) / (81/80)^{1/4}$	459.76	76	76
B 4	$((3f_{E5}) / 4) / (81/80)^{1/4}$	491.93	193	117
C 5	$((2f_{G5}) / 3) (81/80)^{1/4}$	526.36	310	117
C# 5	$((3f_{F\#5}) / 4) / (81/80)^{1/4}$	550	386	76
D 5	$((2f_{A5}) / 3) (81/80)^{1/4}$	588.49	503	117
D# 5	$((3f_{G\#5}) / 4) / (81/80)^{1/4}$	614.92	579	76
E 5	$((3f_{A4}) / 2) / (81/80)^{1/4}$	657.95	697	117
F 5	$((3f_{A\#4}) / 2) / (81/80)^{1/4}$	687.5	773	76
F# 5	$((3f_{B4}) / 2) / (81/80)^{1/4}$	735.61	890	117
G 5	$((4f_{D5}) / 3) (81/80)^{1/4}$	787.1	1007	117
G# 5	$((3f_{C\#5}) / 2) / (81/80)^{1/4}$	822.44	1083	76
A 5	$2f_{A4}$	880	1200	117

The “wolf ” fifth here occurs between the F and the C, but of course is dependent on what pitch is used as the reference tone. The position of the wolf fifth in the scale can be controlled by the point at which pitch frequencies stop being calculated as fifths from the reference tone, and begin being calculated as fourths from a just octave above the reference tone. With the reference tone as the tonic key, the position of the wolf fifth as it is here allows for 7 modulations around a cycle of fifths before it is encountered as part of a tonic chord, but only allows for 2 modulations around a cycle of fourths.

III. 12 tone equal temperament

MIDI note	Equation	Alternative equation	Frequency (Hz) (to 2 decimal places)	Cents (from A4)	Cents (from previous note)
A 4	n/a	n/a	440	0	n/a
A# 4	$(^{12}\sqrt{2})f_{A4}$	$2^{1/12}f_{A4}$	466.16	100	100
B 4	$(^{12}\sqrt{2})f_{A\#4}$	$2^{2/12}f_{A4}$	493.88	200	100
C 5	$(^{12}\sqrt{2})f_{B4}$	$2^{3/12}f_{A4}$	523.25	300	100
C# 5	$(^{12}\sqrt{2})f_{C5}$	$2^{4/12}f_{A4}$	554.37	400	100
D 5	$(^{12}\sqrt{2})f_{C\#5}$	$2^{5/12}f_{A4}$	587.33	500	100
D# 5	$(^{12}\sqrt{2})f_{D5}$	$2^{6/12}f_{A4}$	622.25	600	100
E 5	$(^{12}\sqrt{2})f_{D\#5}$	$2^{7/12}f_{A4}$	659.26	700	100
F 5	$(^{12}\sqrt{2})f_{E5}$	$2^{8/12}f_{A4}$	698.46	800	100
F# 5	$(^{12}\sqrt{2})f_{F5}$	$2^{9/12}f_{A4}$	739.99	900	100
G 5	$(^{12}\sqrt{2})f_{F\#5}$	$2^{10/12}f_{A4}$	783.99	1000	100
G# 5	$(^{12}\sqrt{2})f_{G5}$	$2^{11/12}f_{A4}$	830.61	1100	100
A 5	$(^{12}\sqrt{2})f_{G\#5}$	$2f_{A4}$	880	1200	100

$2^{1/12}$ is equal to $^{12}\sqrt{2}$. An equation that allows for the calculation of pitch frequencies in any equal temperament can be expressed as:

$f_n = R^{n/d}f_1$ where f_n = the frequency of scale degree n

R = the frequency ratio of the interval to be equally divided

n = scale degree

d = the number of equal divisions

and f_1 = the frequency of the reference tone

For example, the frequency of the third chromatic pitch from A at 440 Hz in a 4 tone equal division of a just perfect fifth could be given by:

$$f_3 = 1.5^{3/4}440$$

$= 596.3773224$ where the reference tone is A at 440 Hz

IV. Diatonic just intonation

MIDI note	**Equation**	**Alternative equation**	**Frequency (Hz) (to 2 decimal places)**	**Cents (from A4)**	**Cents (from previous note)**
A 4	n/a	n/a	440	0	n/a
B 4	$((6f_{G\#5}) / 5) / 2$	$(9f_{A4})/ 8$	495	204	204
C# 5	$(5f_{A4}) / 4$	$(10f_{B4}) / 9$	550	386	182
D 5	$4f_{F\#5} / 5$	$(16f_{C\#5}) / 15$	586.67	498	112
E 5	$(6f_{C\#5}) / 5$	$(9f_{D5})/ 8$	660	702	204
F# 5	$2((5f_{A4}) / 6)$	$(10f_{E5}) / 9$	733.33	884	182
G# 5	$(5f_{E5}) / 4$	$(9f_{F\#5})/ 8$	825	1088	204
A 5	$2f_{A4}$	$(16f_{G\#5}) / 15$	880	1200	112

V. Additive Csound Instrument

```
; Additive Instrument, by Nicholas Hender, 1997.

; This instrument supports up to 21 partials. Attack and sustain
; times are the same for each partial, but different decay times
; can be specified, although the decay time of each partial is
; directly proportional to the partial's amplitude. This could be
; somewhat restrictive, but saves time and ensures a fairly
; "natural" sounding decay.


sr=44100    ; header for final "draft"
kr=4410
ksmps=10
nchnls=2

;sr=32000   ; faster header for "rough drafts"
;kr=400
;ksmps=80
;nchnls=2

; "P" fields:

; p1  = instrument number
; p2  = start time
; p3  = duration
; p4  = amplitude multiplier (final adjustment)
; p5  = pan: 0=left, 0.5=centre, 1=right


instr 1


; more "P" fields:

iatt = p6         ; p6 = attack time
isus = p7         ; p7 = sustain time
idec = p8         ; p8 = decay time
idamp = 0.048     ; damping (i.e. release) time
iamp = 32768      ; partial 1 (i.e. fundamental) amplitude
kpitch = p9       ; p9  = partial 1 (i.e. fundamental) frequency (Hz)

iamp2 = p10       ; p10 = partial 2 amplitude ratio
kfrq2 = p11       ; p11 = partial 2 frequency ratio

iamp3 = p12       ; p12 = partial 3 amplitude ratio
kfrq3 = p13       ; p13 = partial 3 frequency ratio

iamp4 = p14       ; p14 = partial 4 amplitude ratio
kfrq4 = p15       ; p15 = partial 4 frequency ratio

iamp5 = p16       ; p16 = partial 5 amplitude ratio
kfrq5 = p17       ; p17 = partial 5 frequency ratio

iamp6 = p18       ; p18 = partial 6 amplitude ratio
kfrq6 = p19       ; p19 = partial 6 frequency ratio

iamp7 = p20       ; p20 = partial 7 amplitude ratio
kfrq7 = p21       ; p21 = partial 7 frequency ratio

iamp8 = p22       ; p22 = partial 8 amplitude ratio
kfrq8 = p23       ; p23 = partial 8 frequency ratio

iamp9 = p24       ; p24 = partial 9 amplitude ratio
```

```
kfrq9 = p25        ; p25 = partial 9 frequency ratio

iamp10 = p26       ; p26 = partial 10 amplitude ratio
kfrq10 = p27       ; p27 = partial 10 frequency ratio

iamp11 = p28       ; p28 = partial 11 amplitude ratio
kfrq11 = p29       ; p29 = partial 11 frequency ratio

iamp12 = p30       ; p30 = partial 12 amplitude ratio
kfrq12 = p31       ; p31 = partial 12 frequency ratio

iamp13 = p32       ; p32 = partial 13 amplitude ratio
kfrq13 = p33       ; p33 = partial 13 frequency ratio

iamp14 = p34       ; p34 = partial 14 amplitude ratio
kfrq14 = p35       ; p35 = partial 14 frequency ratio

iamp15 = p36       ; p36 = partial 15 amplitude ratio
kfrq15 = p37       ; p37 = partial 15 frequency ratio

iamp16 = p38       ; p38 = partial 16 amplitude ratio
kfrq16 = p39       ; p39 = partial 16 frequency ratio

iamp17 = p40       ; p40 = partial 17 amplitude ratio
kfrq17 = p41       ; p41 = partial 17 frequency ratio

iamp18 = p42       ; p42 = partial 18 amplitude ratio
kfrq18 = p43       ; p43 = partial 18 frequency ratio

iamp19 = p44       ; p44 = partial 19 amplitude ratio
kfrq19 = p45       ; p45 = partial 19 frequency ratio

iamp20 = p46       ; p46 = partial 20 amplitude ratio
kfrq20 = p47       ; p47 = partial 20 frequency ratio

iamp21 = p48       ; p48 = partial 21 amplitude ratio
kfrq21 = p49       ; p49 = partial 21 frequency ratio


; oscillators and their corresponding envelopes:

kenv1       expseg    iamp, iatt+isus, iamp, idec, 1
apart1      oscil     kenv1, kpitch, 1

kenv2       expseg    iamp*iamp2, iatt+isus, iamp*iamp2, idec*iamp2, 1
apart2      oscil     kenv2, kpitch*kfrq2, 1

kenv3       expseg    iamp*iamp3, iatt+isus, iamp*iamp3, idec*iamp3, 1
apart3      oscil     kenv3, kpitch*kfrq3, 1

kenv4       expseg    iamp*iamp4, iatt+isus, iamp*iamp4, idec*iamp4, 1
apart4      oscil     kenv4, kpitch*kfrq4, 1

kenv5       expseg    iamp*iamp5, iatt+isus, iamp*iamp5, idec*iamp5, 1
apart5      oscil     kenv5, kpitch*kfrq5, 1

kenv6       expseg    iamp*iamp6, iatt+isus, iamp*iamp6, idec*iamp6, 1
apart6      oscil     kenv6, kpitch*kfrq6, 1

kenv7       expseg    iamp*iamp7, iatt+isus, iamp*iamp7, idec*iamp7, 1
apart7      oscil     kenv7, kpitch*kfrq7, 1

kenv8       expseg    iamp*iamp8, iatt+isus, iamp*iamp8, idec*iamp8, 1
apart8      oscil     kenv8, kpitch*kfrq8, 1

kenv9       expseg    iamp*iamp9, iatt+isus, iamp*iamp9, idec*iamp9, 1
```

```
apart9      oscil     kenv9, kpitch*kfrq9, 1

kenv10      expseg    iamp*iamp10, iatt+isus, iamp*iamp10, idec*iamp10, 1
apart10     oscil     kenv10, kpitch*kfrq10, 1

kenv11      expseg    iamp*iamp11, iatt+isus, iamp*iamp11, idec*iamp11, 1
apart11     oscil     kenv11, kpitch*kfrq11, 1

kenv12      expseg    iamp*iamp12, iatt+isus, iamp*iamp12, idec*iamp12, 1
apart12     oscil     kenv12, kpitch*kfrq12, 1

kenv13      expseg    iamp*iamp13, iatt+isus, iamp*iamp13, idec*iamp13, 1
apart13     oscil     kenv13, kpitch*kfrq13, 1

kenv14      expseg    iamp*iamp14, iatt+isus, iamp*iamp14, idec*iamp14, 1
apart14     oscil     kenv14, kpitch*kfrq14, 1

kenv15      expseg    iamp*iamp15, iatt+isus, iamp*iamp15, idec*iamp15, 1
apart15     oscil     kenv15, kpitch*kfrq15, 1

kenv16      expseg    iamp*iamp16, iatt+isus, iamp*iamp16, idec*iamp16, 1
apart16     oscil     kenv16, kpitch*kfrq16, 1

kenv17      expseg    iamp*iamp17, iatt+isus, iamp*iamp17, idec*iamp17, 1
apart17     oscil     kenv17, kpitch*kfrq17, 1

kenv18      expseg    iamp*iamp18, iatt+isus, iamp*iamp18, idec*iamp18, 1
apart18     oscil     kenv18, kpitch*kfrq18, 1

kenv19      expseg    iamp*iamp19, iatt+isus, iamp*iamp19, idec*iamp19, 1
apart19     oscil     kenv19, kpitch*kfrq19, 1

kenv20      expseg    iamp*iamp20, iatt+isus, iamp*iamp20, idec*iamp20, 1
apart20     oscil     kenv20, kpitch*kfrq20, 1

kenv21      expseg    iamp*iamp21, iatt+isus, iamp*iamp21, idec*iamp21, 1
apart21     oscil     kenv21, kpitch*kfrq21, 1


; the additive part:

aa = apart1 + apart2 + apart3 + apart4 + apart5 + apart6 + apart7
ab = apart8 + apart9 + apart10 + apart11 + apart12 + apart13 + apart14
ac = apart15 + apart16 + apart17 + apart18 + apart19 + apart20 + apart21
ad = aa + ab + ac


; a global envelope to enable the cut off of a note before the
; expiration of its decay time:

kenv  linen 1, iatt, p3, idamp
asnd=(ad*kenv)


; output assignment:

aoutl = (asnd*p4)*(1-p5)      ; stereo for final draft
aoutr = (asnd*p4)*p5
outs  aoutl, aoutr

;aout = (asnd*p4)+(p5-p5)     ; mono for rough drafts, p5 is dummy
;out  aout


endin
```

VI. The harmonic scale

MIDI note	Equation	Frequency (Hz) (to 2 decimal places)	Cents (from A4)	Cents (from previous note)
A 4	n/a	440	0	n/a
A# 4	$(17f_{A4}) / 16$	467.5	105	105
B 4	$(9f_{A4}) / 8$	495	204	99
C 5	$(19f_{A4}) / 16$	522.5	298	94
C# 5	$(5f_{A4}) / 4$	550	386	89
D 5	$(21f_{A4}) / 16$	577.5	471	84
D# 5	$(11f_{A4}) / 8$	605	551	81
E 5	$(3f_{A4}) / 2$	660	702	151
F 5	$(13f_{A4}) / 8$	715	841	139
F# 5	$(27f_{A4}) / 16$	742.5	906	65
G 5	$(7f_{A4}) / 4$	770	969	63
G# 5	$(15f_{A4}) / 8$	825	1088	119
A 5	$2f_{A4}$	880	1200	112

VII. Sethares's BASIC program for calculating dissonance curves

```
'       A Microsoft QBasic program by William Sethares (1993).
'       This program calculates the co-ordinates for the
'       "dissonance curve" corresponding to any given timbre.

'       Additional loop so that co-ordinates are displayed one
'       screen at a time, and re-scaling of dissonance values, both
'       added by Nicholas Hender (1997).

DIM freq(10), amp(10), g(10), diss(1500)

'Here are the variables you must set:
'numf=number of frequencies in timbre
'freq(i)=frequency value of ith partial
'amp(i)=amplitude of ith partial
'max = 10 times the reciprocal ofthe most dissonant value obtained
```

```
'Setting max after the program has run
'scales the dissonance values between 0 and 10.

numf = 7
freq(1) = 500: freq(2) = 1000: freq(3) = 1500: freq(4) = 2000
freq(5) = 2500: freq(6) = 3000: freq(7) = 3500
amp(1) = 10: amp(2) = 8.8: amp(3) = 7.7: amp(4) = 6.8
amp(5) = 6: amp(6) = 5.3: amp(7) = 4.6

max = .230851449

'loop through all intervals from startint to endint

dstar = .24: s1 = .0207: s2 = 18.96: c1 = 5!: c2 = -5!
a1 = -3.51: a2 = -5.75: index = -1
startint = .79: endint = .99: inc = .01

FOR z = 1 TO 6
    PRINT " Interval      Dissonance"
    startint = startint + .21
    endint = endint + .21

    FOR alpha = startint TO endint STEP inc
        index = index + 1: d = 0

            FOR k = 1 TO numf
                g(k) = alpha * freq(k)
            NEXT k

            'calculate dissonance between f and alpha*f
            FOR i = 1 TO numf

               FOR j = 1 TO numf
                 IF amp(i) < amp(j) THEN lij = amp(i) ELSE lij = amp(j)
                 IF g(j) < freq(i) THEN fmin = g(j) ELSE fmin = freq(i)
                 s = dstar / (s1 * fmin + s2): fdif = ABS(g(j) - freq(i))
                 arg1 = a1 * s * fdif: arg2 = a2 * s * fdif
                 IF arg1 < -88 THEN exp1 = 0 ELSE exp1 = EXP(arg1)
                 IF arg2 < -88 THEN exp2 = 0 ELSE exp2 = EXP(arg2)
                 dnew = lij * (c1 * exp1 + c2 * exp2): d = d + dnew
               NEXT j

            NEXT i

        diss(index) = d
        y = max * d
        PRINT alpha, y
    NEXT alpha

    PRINT " Press any key, and then F5 to continue."
   STOP

NEXT z

STOP
```

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